Applying Newton's Law of Cooling When the Environment Keeps Changing Temperature, Such as in Stratospheric Ballooning Missions

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Newton's Law of Cooling describes how a "small" system, such as a thermometer, comes to thermal equilibrium with a "large" system, such as its environment, as a function of time. It is typically applied when the environment is in thermal equilibrium and the conditions are such that the thermal decay time for the thermometer is a constant. Neither of these conditions are met when measuring environmental (i.e. atmospheric) temperature using a thermometer mounted in a payload lofted into the stratosphere under weather balloons. In this situation the thermometer is in motion so it encounters layer after layer of atmosphere which differ in temperature, and the changing environmental conditions can influence the thermal decay time "constant" for the thermometer as well. We have used Newton's Law of Cooling in spreadsheet-based computer simulations to explore how thermometer readings react under these conditions and to better-understand how logger temperature records from stratospheric balloon flights, during both ascent (relatively slow) and descent (much faster, especially at altitude,) are related to actual environmental temperatures at various altitudes.

I. Nomenclature

dQ/dt	=	rate of heat flow
k	=	constant of proportionality in Newton's Law of Cooling
T	=	object's temperature
T_{env}	=	environmental temperature
τ	=	characteristic decay time constant
t	=	time
R	=	drift rate of thermometer

II. Introduction

Newton's Law of Cooling [1], for use in situations where heat is transferred by convection, states that the rate of heat flow dQ/dt is proportional to the difference an object's temperature T and the environmental temperature T_{env}.

$$dQ/dt = k * (T - T_{env}) \tag{1}$$

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Here the proportionality value *k* depends on heat transfer area and heat transfer efficiency and is usually assumed to be a constant.

If an object that starts at temperature T_0 in an environment of constant temperature T_{env}, this law suggests that the object's temperature *T[t]* will decay exponentially in time toward T_{env} as

$$T[t] = T_{env} + (T_0 - T_{env}) * exp[- t / \tau] \qquad (2)$$

Here the characteristic decay time constant τ is how long it takes for the temperature it get within $1/e = 0.368$ of the final value T_{env}.

III. Excel Simulation Results

An Excel spreadsheet was written to simulate Newton's Law of Cooling using small time steps (much smaller than τ) [2]. Figure 1a shows *T[t]* for two objects which start at different temperatures but tend toward the same environmental temperature with the same decay time constant. The temperature difference between each object and the environment is also plotted. Figure 1b, on the other hand, shows *T[t]* for two objects with the same initial and final temperatures, but which have different decay time constants. An object has come to thermal equilibrium with its environment when its temperature is indistinguishable from that of the environment, within the resolution of the measuring device. For the same decay time constant, an object that starts with a temperature farther from the environmental temperature takes longer to reach equilibrium with the environment. On the other hand, the longer the decay time constant, the longer it takes for an object to transfer heat and hence the longer it takes to reach thermal equilibrium with its environment.

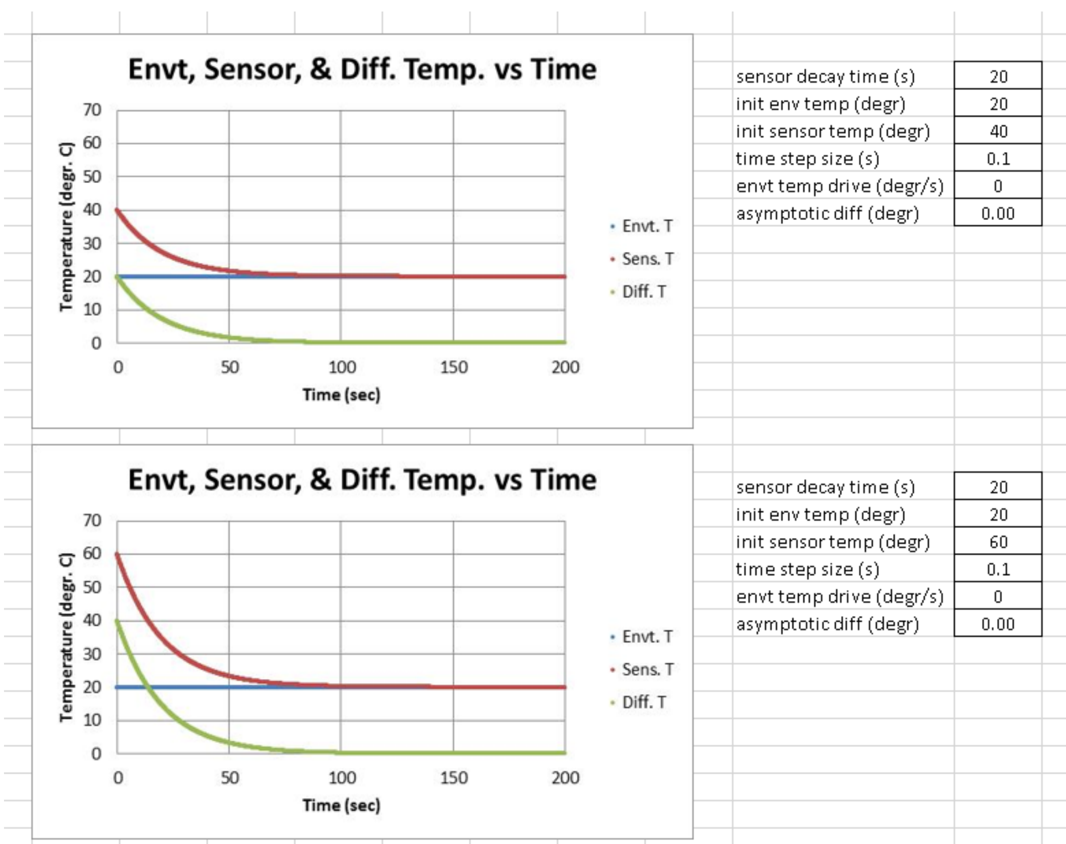

Fig. 1a Comparison of two objects with different initial temperatures but the same thermal decay time constant coming into equilibrium with their environment.

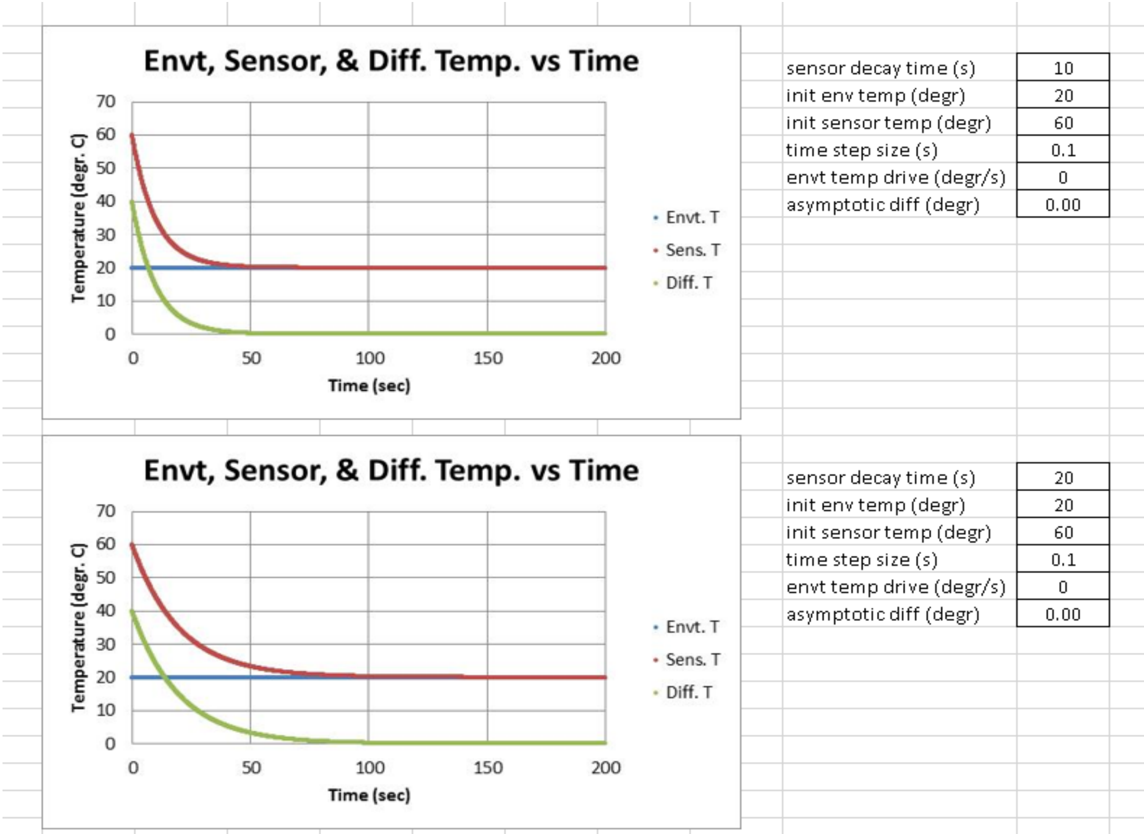

Fig. 1b Comparison of two objects with the same initial temperature but with different thermal decay time constants coming into equilibrium with their environment.

If the object in question is a thermometer and it is being used to measure the environmental temperature, a fair question might be "How long must one wait before the thermometer is actually reporting the environmental temperature?" The answer, as illustrated by the figures above, is "Eventually." The farther apart the initial thermometer temperature is from the environmental temperature and the longer the decay time constant, the longer this will take. But after a few τ's have elapsed the thermometer temperature will be indistinguishable from the environmental temperature.

However if the environmental temperature is constantly changing, such as is the case during stratospheric balloon flights, the answer becomes "Never!" Now the thermometer will "chase" the environmental temperature but will never come into equilibrium with it. The same Excel simulation was used to explore the changing-environmental-temperature situation. The simplest possible situation to consider is one where the environmental temperature changes linearly in time. Figure 2(a) shows how a specific thermometer (with a specific decay time constant) will eventually parallel the environmental temperature with the same offset temperature when the thermometer starts out warmer than or colder than (or even the same temperature as – not shown) the environment. Figure 2(b) shows how two thermometers with different decay time constants behave similarly, though the "faster" thermometer will reach the parallel-temperature condition more quickly and with a smaller temperature offset. Figure 2(c) illustrates how the temperature offset for a given thermometer is directly proportional to the rate at which the environmental temperature is changing. Simply put, Newton's Law of Cooling suggests that a thermometer needs a specific temperature offset from the environmental temperature to drive it to change at a specific rate. Once that temperature offset is reached, the thermometer temperature will maintain that offset rather than continuing to get closer and closer to the environmental temperature.

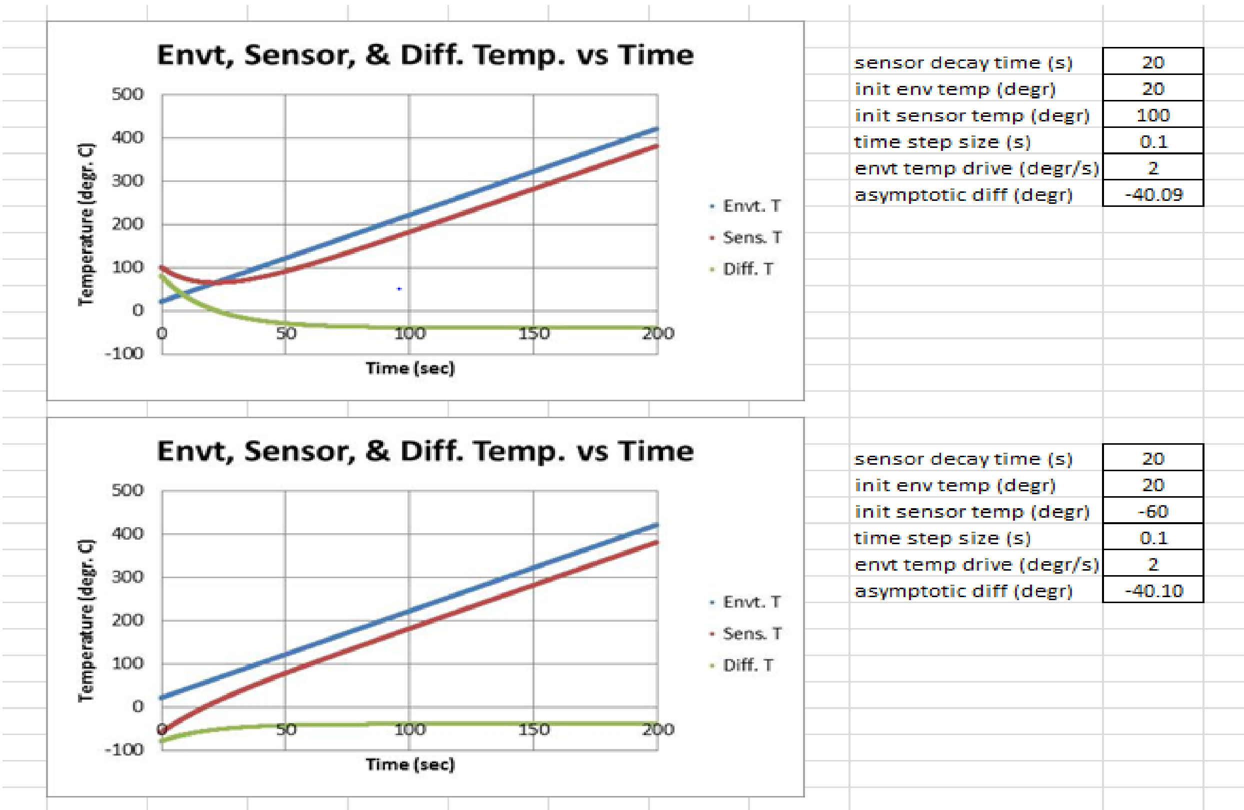

Fig. 2a When the environmental temperature is changing linearly in time, the thermometer reading will ultimately parallel the environmental temperature. The end result is exactly the same whether the thermometer starts out initially warmer than or colder than the environment.

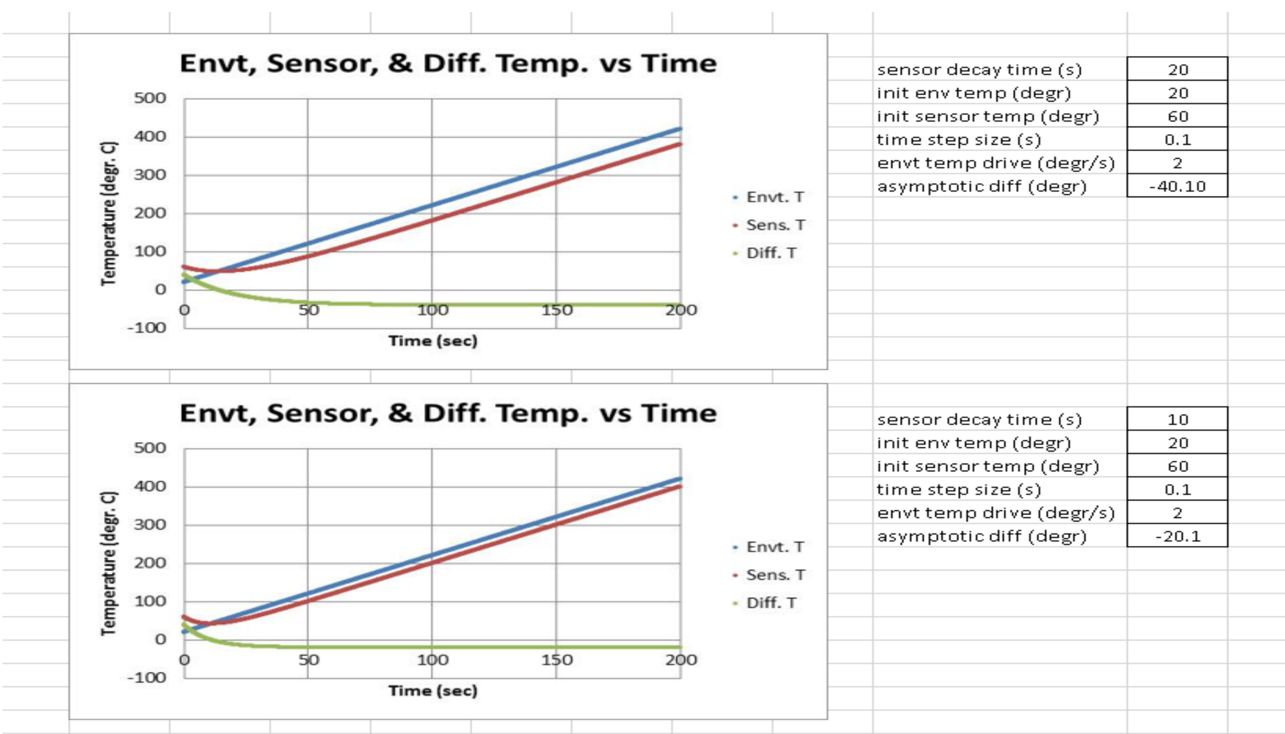

Fig. 2b A thermometer with a shorter decay time constant is "more responsive" and ends up paralleling the environmental temperature more quickly and with a smaller temperature offset.

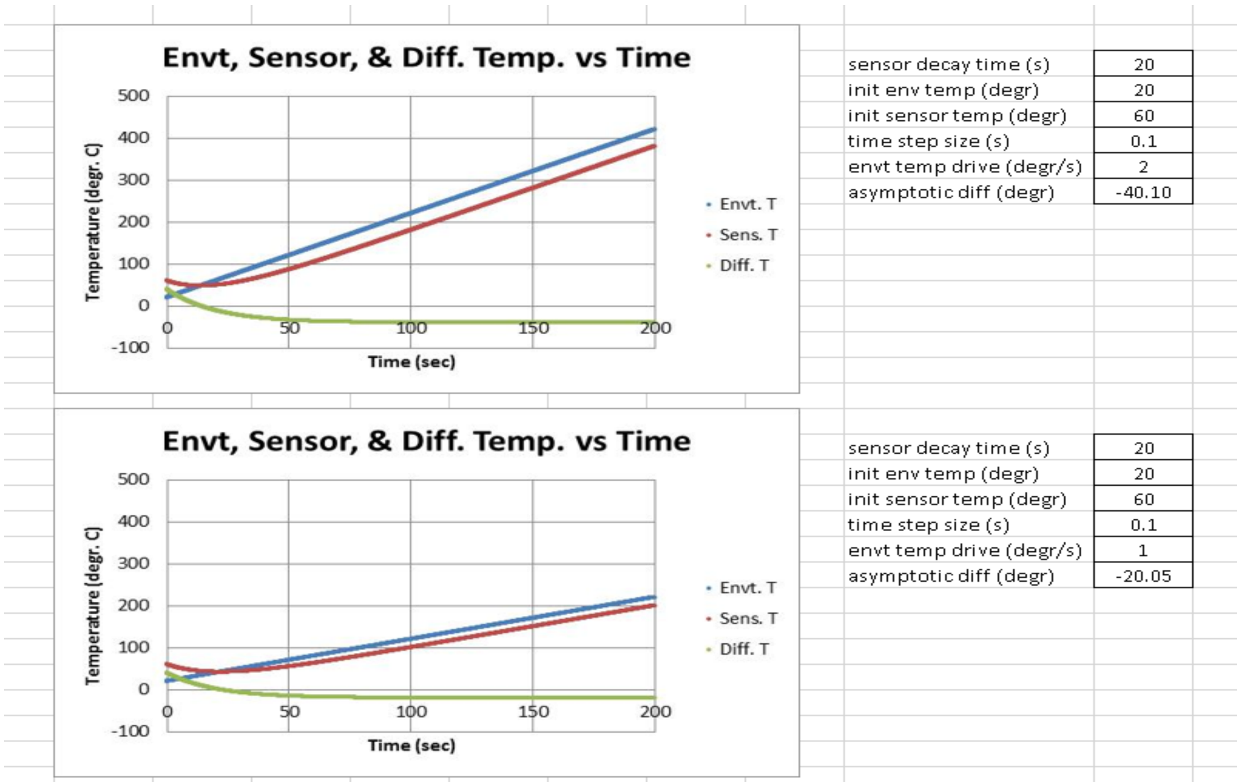

Fig. 2c For a thermometer with a specific decay time constant, the size of the temperature offset between the environment and the thermometer reading depends linearly on environmental temperature change rate.

Even though the thermometer never reports the true environmental temperature when the environmental temperature changes linearly with time, except if the two temperature curves happen to cross, the thermometer can still be used to determine the environmental temperature. To do this one must first document the decay time constant τ for the thermometer by doing exponential time decay fits of *T[t]* as the thermometer comes into equilibrium with a fixed-temperature environment. Then, when the thermometer is in contact with an environment whose temperature is changing linearly in time, the drift rate *R* that the thermometer ultimately reaches will be the same rate that the environmental temperature is changing with time. The time derivative of *T[t]* above, in the limit where t approaches zero, is $dT/dt = \Delta T * (- 1 / \tau)$. If this is to equal *R*, then the temperature offset must be $\Delta T = - R \tau$, a particularly simple result. Note: slight discrepancies from this result arise from the finite step size of the simulation. As anticipated above, the temperature offset grows linearly with the environmental temperature drift rate *R*. The minus sign indicates that the thermometer temperature always lags the environmental temperature. If *R* is positive (i.e. the environment is warming), then ΔT will be negative (i.e. the thermometer will always be behind (i.e. cooler than) the environment). Conversely, if the environmental temperature is going down then *R* will be negative so ΔT will be positive (i.e. the thermometer will again be behind (i.e. now warmer than) the environment).

An extension of the Excel spreadsheet allows us to simulate the response of a thermometer to the 5 phases of a typical stratospheric balloon flight: (1) temperature decreasing relatively slowly during ascent through the troposphere, (2) temperature increasing relatively slowing during ascent into the stratosphere, (3) temperature decreasing relatively quickly during (post-burst) descent back down to the tropopause, (4) temperature increasing relatively quickly during descent to the ground, and (5) temperature not changing once the payloads are back on the ground. Figure 3 shows how two thermometers with different decay time values, assumed to be constant throughout the flight, would respond to this actual temperature profile if all the environmental variations were linear in time (which is actually not the case). Simply put, the thermometers report a "distorted/delayed" version of the true environmental temperature profile. As before, the "faster" thermometer (i.e. the one with the shorter decay time constant) is more responsive and follows the environmental temperature changes more exactly, with smaller temperature offsets.

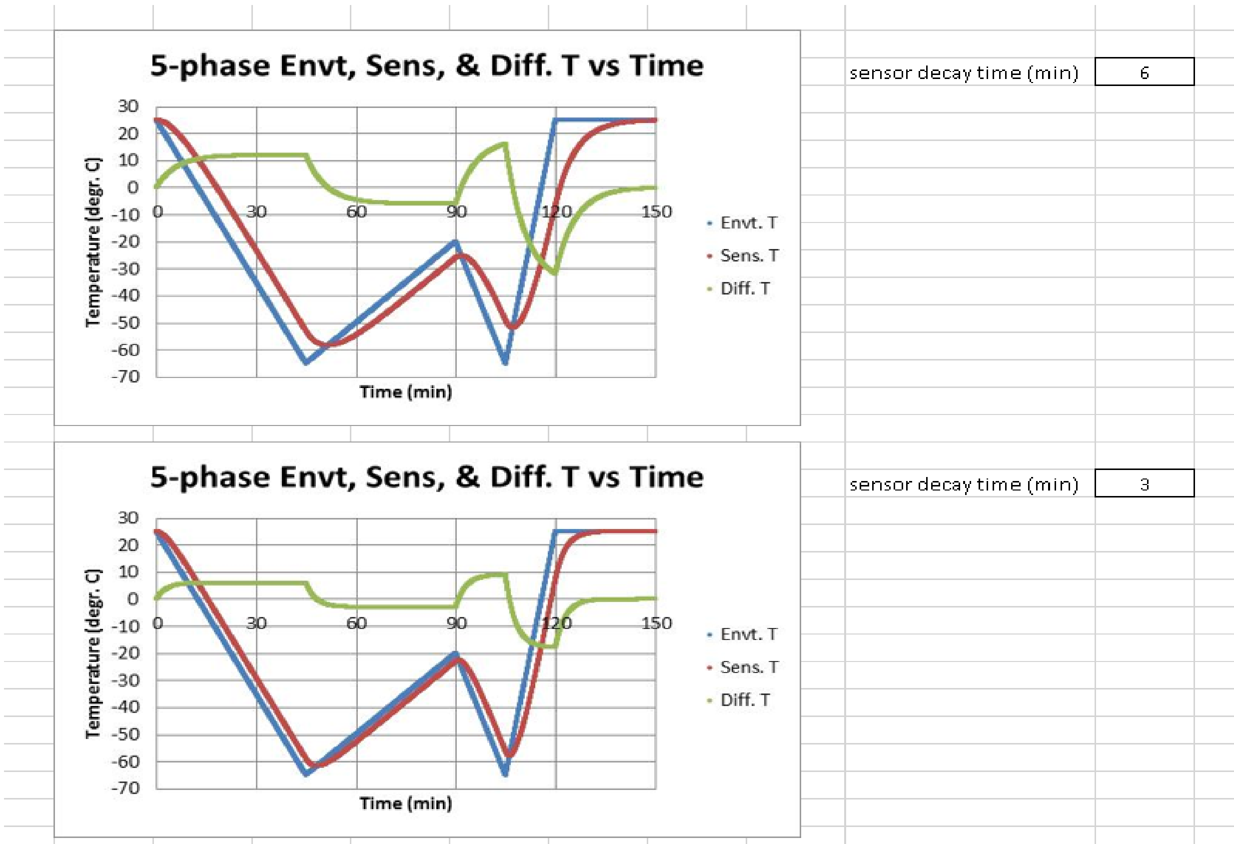

Fig. 3 Simulations of how thermometers with two different decay time constants would react to the 5 phases of temperature change (assumed to be linear) during a stratospheric balloon flight.

The ultimate goal of this exploration is basically do this backwards – to determine the actual environmental temperature from stratospheric balloon flights using actual thermometer-reported temperature profiles such as those shown in the next section. "Correcting" the temperatures will require experimentally-determined knowledge of the decay time constants for thermometers which might not end up being constant under all the conditions encountered during a stratospheric balloon flight where the atmospheric pressure falls dramatically even as the temperature fluctuates, depending on the layer of the atmosphere the thermometer is passing through.

IV. Preliminary Experimental Results

The high-altitude ballooning teams at the University of Minnesota – Twin Cities and at St. Catherine University have made environmental temperature measurements during many stratospheric balloon flights using (a) Onset Computer's Air/Water/Soil 1-foot temperature sensors for HOBO data loggers [3], (b) Neulog's wide-range temperature (thermocouple) sensors [4], and (c) Maxim's (Arduino-logged) DS18B20 Dallas 1-Wire digital temperature sensors [5].

To characterize the time decay constants for all 3 types of thermometers at the same time, a "triple-temperature" device was built which included a HOBO, a Neulog module chain, and an Arduino Uno, with the three sensors listed above mounted within 1.5 centimeters of each other (see Fig. 4a and close-up in Fig. 4b).

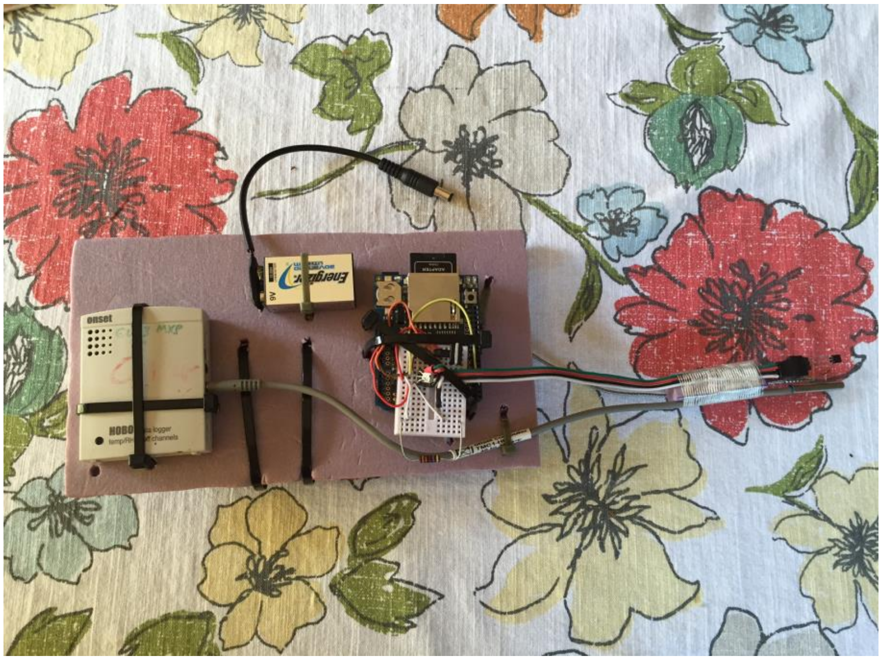

Fig. 4a Triple-temperature logger. HOBO and Arduino loggers visible; Neulog modules on underside.

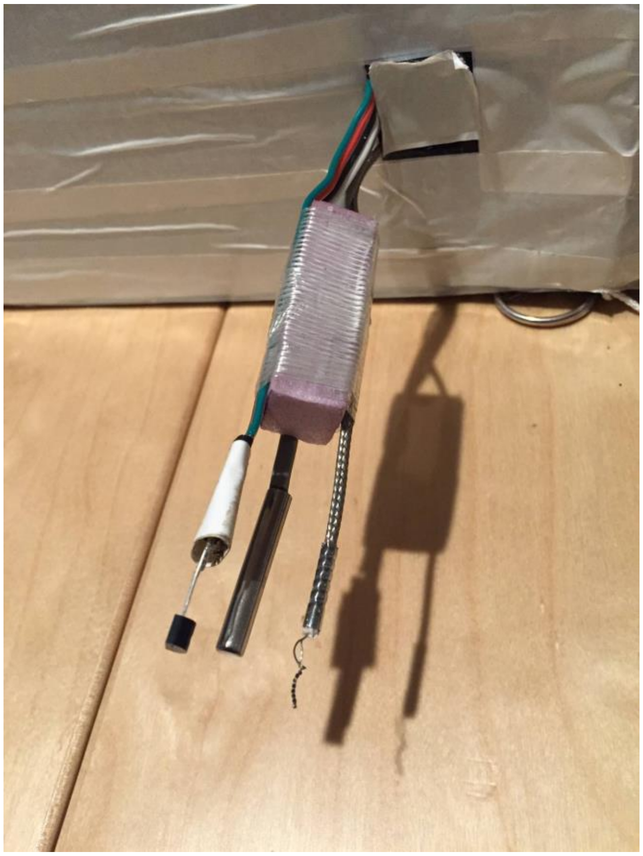

Fig. 4b Close-up view of the triple-temperature tip with DS18B20 Dallas 1-Wire (left), HOBO temperature sensor (center), and Neulog thermocouple (right), all within 1.5 cm of each other.

This device was moved between a deep freeze and home-temperature air multiple times to characterize the decay time constant for each type of sensor. The time decay constant results under standard atmospheric pressure conditions were as follows:

$$\tau_{HOBO} = 223 \text{ seconds}; \;\; \tau_{Neulog} = 23 \text{ seconds}; \;\; \tau_{Dallas} = 190 \text{ seconds}$$

To determine if τ values change with environmental pressure – we hypothesized that the sensors might react more slowly (i.e. have larger τ values) at reduced pressure – the device was "slim-mounted" on a sled that could fit into a 3-inch diameter pvc tube which was then evacuated using a vacuum pump. The two ends of the tube were held at different temperatures by covering one end in ice cubes. The sled started at the cold end but then was slid to the warm end without breaking the vacuum seal by tipping the tube – one could hear it slide through the tube easily. Figure 5a shows the slim-mounted version and Fig. 5b shows the pvc experimental set-up for the reduced-pressure test.

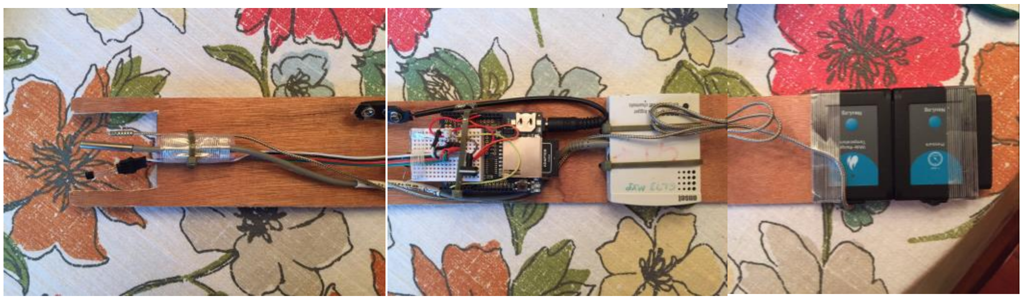

Fig. 5a "Slim mount" of triple-temperature logger for reduced-pressure testing.

Fig. 5b The 3-inch diameter PCV pipe used for the reduced-pressure test. The pipe was evacuated through the vacuum hose attached to the central coupler. The far end of the pipe was kept at ice temperature.

Plots of the reduced-pressure test results (all versus time) are shown in Fig. 6a – Pressure, Fig. 6b – Arduino (Dallas) temperature, and Fig. 6c – HOBO temperature. The Neulog thermocouple sensor failed mid-test , so no useful data was forthcoming. The time decay constant results in reduced pressure are listed below. Both time constants were longer than the values at full atmospheric pressure: 66% longer for the HOBO temperature sensor and 24% longer for the Dallas temperature sensor. The pressure was reduced to 1.2 psi by the vacuum pump. The time decay constants were determined during the warming period which all occurred at low pressure.

τ_{HOBO} = 370 sec; τ_{Neulog} = TBA (sensor failed); τ_{Dallas} = 236 sec

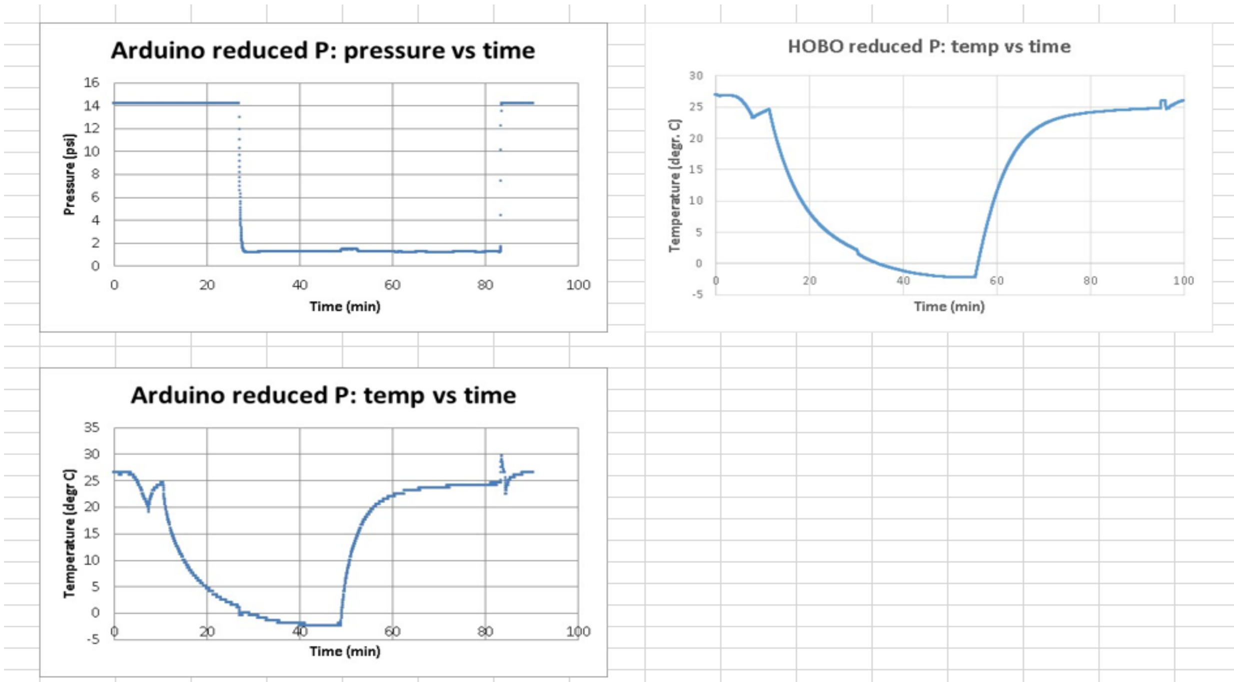

Fig. 6a (upper left) Pressure vs Time during the reduced-pressure test.
Fig. 6b (upper right) HOBO sensor cooling then re-warming, the latter under reduced pressure.
Fig. 6c (lower left) Arduino-logged Dallas sensor cooling then re-warming under low pressure.

The triple-temperature device has been flown on two stratospheric balloon missions to date. Figure 7a-d shows temperature versus time graphs for the 3 types of temperature sensors (plus one pressure versus time graph) from one flight. Future work includes trying to apply simulation capabilities and experimental knowledge of decay time constants for the various sensors at atmospheric pressure and at the one reduced pressure tested to determine what single/actual atmospheric temperature profile is simultaneously consistent with all the plots below. Additional ground tests at other reduced-pressure values and/or at even lower temperatures are also being considered.

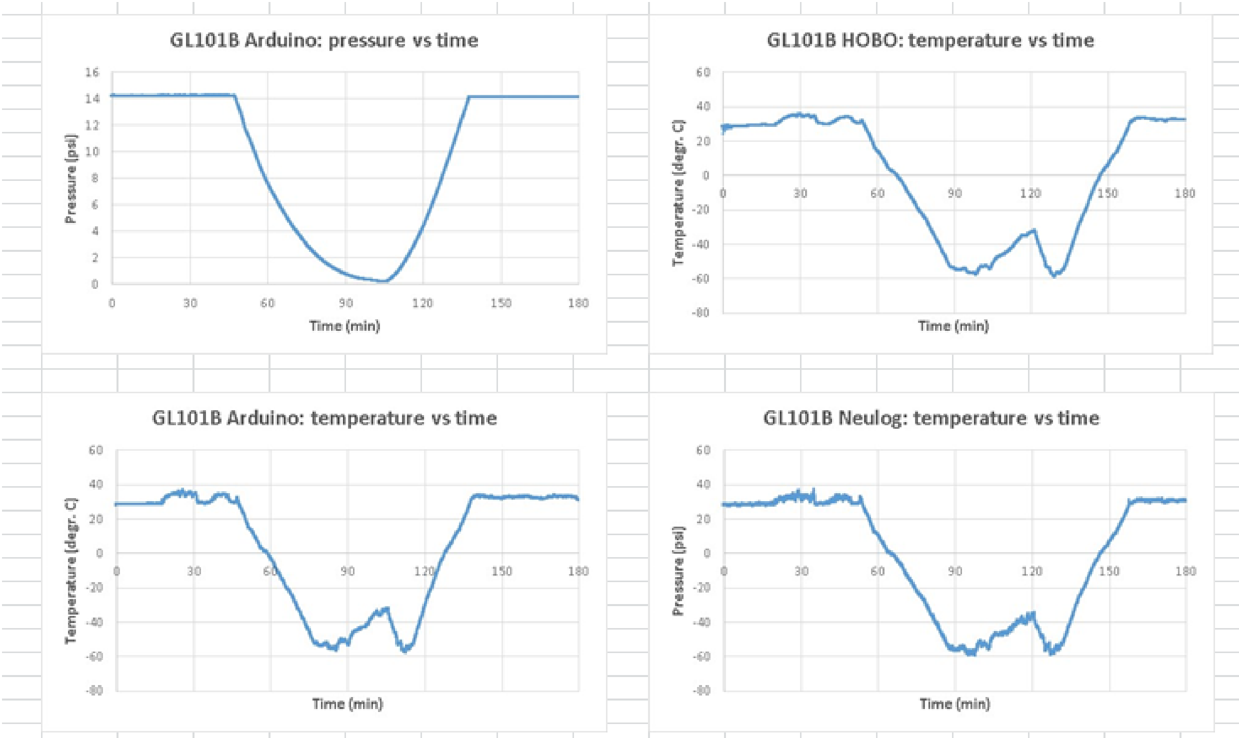

Fig. 7a (upper left) Pressure versus time from a stratospheric balloon mission.
Fig. 7b, 7c, 7d (upper right, lower left, lower right) Temperature versus time from a stratospheric balloon mission. These are actual temperature records from 3 different temperature sensors on one flight. No decay time adjustments have been made to the data.

V. Conclusion

Implementing Newton's Law of Cooling using an Excel spreadsheet has allowed us to apply it to situations where the environmental temperature is not constant. This has given us insight into differences between thermometer readings and various time-dependent profiles of actual environmental conditions, with an eye toward ultimately reaching conclusions about atmospheric temperatures during high-altitude balloon flights using "distorted/delayed" temperature records due to thermometer decay time constants. Comparison of thermometer decay times at atmospheric pressure and at a reduced pressure near 1 psi suggest that these decay times are not in fact constant over the wide range of pressures encountered during stratospheric ballooning missions, further complicating analysis of (and correction of) logged temperature data.

VI. Acknowledgments

This work was supported by NASA's MN Space Grant Consortium, the University of Minnesota – Twin Cities, and Saint Catherine University.

VII. References

[1] Bird, R., Stewart, W., Lightfoot, E. (1960). *Transport Phenomena*. New York, NY: John Wiley and Sons Inc.
[2] Spreadsheet available upon request from the first author – flate001@umn.edu.
[3] http://www.onsetcomp.com/products/sensors/tmc1-hd
[4] https://neulog.com/wide-range-temperature/
[5] https://datasheets.maximintegrated.com/en/ds/DS18B20.pdf