Improving Radio Astronomy Using High Altitude Balloons as Calibration Sources

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The purpose of our research was to develop the theory for using high-altitude balloons as artificial guide stars for ground-based radio telescopes. Current methods of ground-based radio astronomy are limited by the necessity of having a guide star within a small angular distance from the source of interest, which is used to correct distortions of the signal caused by the atmosphere. This drastically limits the set of observable stars. If, however, high-altitude balloons equipped with radio transmitters were used to accomplish this task, it would open up a range of new possibilities for ground-based radio observatories. There are two major problems involved. The first is determining the set of all locations where the balloon is within the range of allowable angles. The second is calculating the angle that the telescope makes between the balloon and the source as measured from the telescope location. We present the results of theoretical analysis of the problem as well as the initial results of software tools that have been developed to test this hypothesis.

Nomenclature

α = balloon angle
R_E = radius of the Earth
O = origin

T = telescope vector in Cartesian coordinates
T_{alt} = telescope altitude
T_{lat} = telescope latitude
T_{long} = telescope longitude
$T_\rho = T_{alt} + R_E$

B = balloon vector in Cartesian coordinates
B_{alt} = balloon altitude
B_{lat} = balloon latitude
B_{long} = balloon longitude
$B_\rho = B_{alt} + R_E$

S = source vector in Cartesian coordinates
δ = source declination
HA = source hour angle

TB = vector from the telescope to the balloon in Cartesian coordinates
TS = vector from the telescope to the source in Cartesian coordinates

$R_y(\delta)$ = rotation by source declination about y-axis
$R_z(HA)$ = rotation by source hour angle about z-axis
U = a point in the initial frame
V = a point in the rotated and translated frame

g(t) = signal sent by balloon
f(t) = signal measured by satellite

I. Introduction

For the length of this paper, we will be focusing on radio astronomy. Ground-based observatories are currently limited by the distorting effects of the Earth's atmosphere. While methods exist to correct these aberrations, we would like to propose a superior method utilizing high-altitude balloons.

II. Preliminary Information on Astronomy

There are three preliminary concepts that are critical to understanding this material. First, the Earth's atmosphere blocks out much of the light coming from space. Only a few bands of the electromagnetic spectrum reach the Earth's surface. The two bands that do are the visible and radio bands. These are what astronomers study from ground-based observatories.

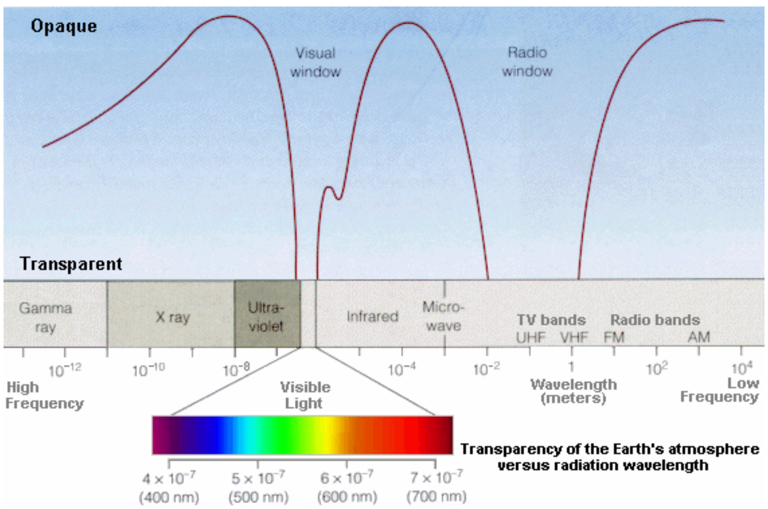

Figure 1: Atmospheric windows.

Another important concept to be aware of when discussing telescopes is angular resolution, which is the minimum angle at which two points in an image can be resolved. This value is given by the following equation where D is the diameter of the telescope, λ is the wavelength of light, and θ is the angular resolution.

$$\sin\theta = 1.220\frac{\lambda}{D} \tag{1}$$

Smaller θ values, or a larger angular resolution, make it possible to distinguish between two points that are closer together.

One technique to maximize angular resolution is called aperture synthesis or synthesis imaging. Since radio wavelengths are large in comparison to other waves, the angular resolution of radio telescopes will be smaller. To compensate for this, an array of telescopes is used, which effectively provides a system that has an angular resolution equivalent to that of a telescope with the same diameter as the longest baseline in the array, thereby increasing the angular resolution.

III. Problems Introduced by the Atmosphere

When observing a source through the Earth's atmosphere, there are many factors which affect the image one receives, such as turbulence, temperature and density variations, and the composition of the atmosphere (most of these phenomena occur in the troposphere). This effect, termed atmospheric seeing, is caused by variations of the refractive index of the atmosphere caused by these factors.

When light enters the atmosphere, it is all in the same phase, or coherent. In effect, it reaches the observer at the same time and from the same direction. However, when it passes through a turbulent medium—like the atmosphere—it becomes distorted and is no longer in phase, and thus arrives at different times from different directions.

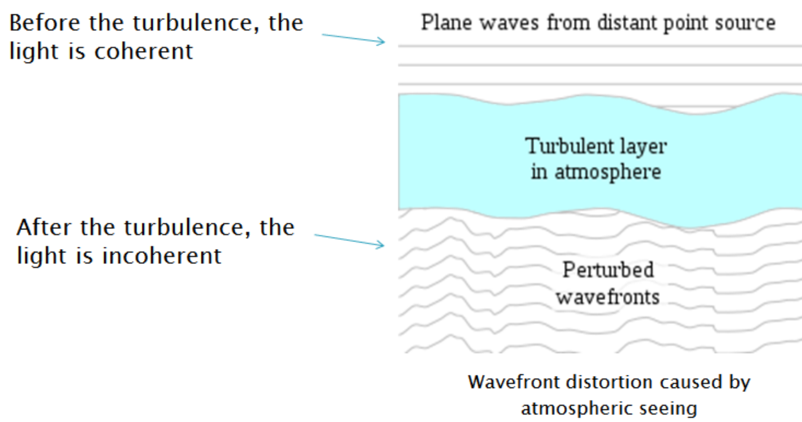

Figure 2: Atmospheric distortions.

IV. Guide Stars

To compensate for this distortion, there must be a bright object within a small angular distance from the source, called a guide star, which provides a signal strong enough to accurately measure the wavefront distortion. This is because the light from the guide star must travel through roughly the same column of air as the light from the source. This limits the set of observable stars to only those which have a bright guide star within a small angular distance (roughly 6 degrees).

To solve this problem for optical telescopes, astronomers use artificial guide stars, generally high-powered lasers, to provide a reference for any star in the sky. Artificial guide stars are only possible, however, for optical wavelengths because only light with short wavelengths can reflect off air molecules and return to the telescope.

One option for an artificial guide star that could be used in radio astronomy would be a satellite. This would seem advantageous because, similar to natural guide stars, the signal sent by the satellite would travel through entire depth of the atmosphere. However, multiple problems arise from this technique. The first problem is that the satellite must be placed in the line of sight from the observatory to the source, which is a complicated and expensive process. The second problem appears in the technique of aperture synthesis. For this technique to be useful, the satellite would have to be within the maximum allowable angle for *all* of the telescopes in the array, which is impossible for some of the largest arrays.

On the other hand, the use of high-altitude balloons would be superior to the use of satellites. An immediate advantage to the use of satellites is that balloons are much easier to operate and are far less expensive. Although the altitude of the balloon is not as great as that of the satellite, the amount of error introduced is negligible, since the troposphere is the main source of atmospheric distortion.

V. Interferometry in Radio Astronomy

In order to correct for atmospheric aberrations, the technique of convolution can be applied. We let $g(t)$ be the known signal sent by the balloon and $f(t)$ be the signal measured by the telescope.

The convolution of f and g is defined as: $$[f * g](a) \equiv \int_0^a f(a-t)\cdot g(t)\,\mathrm{d}t \qquad (2)$$

By maximizing this function, we obtain a value of a, which gives how much f needs to be delayed to be in phase with g. Since the radio dish is divided into an array of pixels, we obtain an array of delay values over the dish which can be used to correct the source signal.

VI. Mathematical Problems

A. The Set of Solutions Problem

There are multiple mathematical problems that must be solved in order to accomplish our task. The first problem is the "set of solutions problem." A desirable piece of information pre-flight would be the set of all locations where the balloon can be and still accomplish its task. Clearly, since all such locations are defined by a maximum angle, the form of the solution will be a cone. We will start with a cone oriented along the x-axis and perform this coordinate transformation:

$$R_z(HA) = \begin{bmatrix} \cos(HA) & -\sin(HA) & 0 \\ \sin(HA) & \cos(HA) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$R_y(\delta) = \begin{bmatrix} \cos(\delta) & 0 & \sin(\delta) \\ 0 & 1 & 0 \\ -\sin(\delta) & 0 & \cos(\delta) \end{bmatrix} \tag{4}$$

$$T = \begin{bmatrix} (T_{alt} + R_E)\cos(T_{long})\sin(90 - T_{lat}) \\ (T_{alt} + R_E)\sin(T_{long})\sin(90 - T_{lat}) \\ (T_{alt} + R_E)\cos(90 - T_{lat}) \end{bmatrix} \tag{5}$$

$$V = R_y(\delta)R_z(-HA)(U - T) \tag{6}$$

This transformation takes a point U and translates it to the location of the telescope on the surface of the earth. Then, it is rotated by the source hour angle, *HA*, about the z-axis. Finally, it is rotated by the source declination, δ, about the new y-axis.

The final solution is a cone whose vertex is the location of the telescope, whose axis is the TS vector, and whose angle is the maximum allowable angle. As previously stated, the balloon can be anywhere in the interior of the cone because the angle it makes will be equal to or less than the maximum angle.

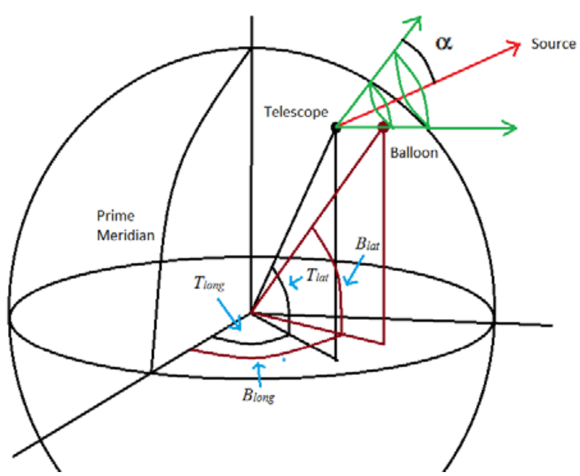

Figure 3: The cone of solutions.

Finally, if we plug everything into one equation, we get the actual formula for the cone:

$$
\begin{aligned}
\alpha = \arccos\Big(& \big(\cos(HA)\sin(\delta)\,\big((B_{alt}+R_E)\cos(B_{long})\cos(B_{lat}) - (T_{alt}+R_E)\cos(T_{long})\cos(T_{lat})\big) + \cos(HA)\cos(\delta)\,\big((B_{alt} \\
& + R_E)\sin(B_{long})\cos(B_{lat}) - (T_{alt}+R_E)\sin(T_{long})\cos(T_{lat})\big) + \sin(HA)\,\big((B_{alt}+R_E)\sin(B_{lat}) - (T_{alt}+R_E)\sin(T_{lat})\big)\big) \Big/ \\
& \Big(\big(\cos(HA)\sin(\delta)\,\big((B_{alt}+R_E)\cos(B_{long})\cos(B_{lat}) - (T_{alt}+R_E)\cos(T_{long})\cos(T_{lat})\big) + \cos(HA)\cos(\delta)\,\big((B_{alt} \\
& + R_E)\sin(B_{long})\cos(B_{lat}) - (T_{alt}+R_E)\sin(T_{long})\cos(T_{lat})\big) + \sin(HA)\,\big((B_{alt}+R_E)\sin(B_{lat}) - (T_{alt}+R_E)\sin(T_{lat})\big)\big)^2 \\
& + \big(-\cos(\delta)\,\big((B_{alt}+R_E)\cos(B_{long})\cos(B_{lat}) - (T_{alt}+R_E)\cos(T_{long})\cos(T_{lat})\big) + \sin(\delta)\,\big((B_{alt}+R_E)\sin(B_{long})\cos(B_{lat}) \\
& - (T_{alt}+R_E)\sin(T_{long})\cos(T_{lat})\big)\big)^2 + \big(-\sin(HA)\sin(\delta)\,\big((B_{alt}+R_E)\cos(B_{long})\cos(B_{lat}) - (T_{alt} \\
& + R_E)\cos(T_{long})\cos(T_{lat})\big) - \sin(HA)\cos(\delta)\,\big((B_{alt}+R_E)\sin(B_{long})\cos(B_{lat}) - (T_{alt}+R_E)\sin(T_{long})\cos(T_{lat})\big) \\
& + \cos(HA)\,\big((B_{alt}+R_E)\sin(B_{lat}) - (T_{alt}+R_E)\sin(T_{lat})\big)\big)^2\Big)^{1/2}\Big)
\end{aligned}
\tag{7}
$$

B. The Angle Problem

The second problem is the "angle problem." Our effort would be fruitless if we could not determine whether or not the balloon was within the range of allowable angles. This can be done easily by computing the dot product of two carefully chosen vectors (TB and TS):

$$
TB = \begin{bmatrix} (B_{alt}+R_E)\cos(B_{long})\sin(90 - B_{lat}) - (T_{alt}+R_E)\cos(T_{long})\sin(90 - T_{lat}) \\ (B_{alt}+R_E)\sin(B_{long})\sin(90 - B_{lat}) - (T_{alt}+R_E)\cos(T_{long})\sin(90 - T_{lat}) \\ (B_{alt}+R_E)\cos(90 - B_{lat}) - (T_{alt}+R_E)\cos(90 - T_{lat}) \end{bmatrix} \tag{8}
$$

$$
TS = \begin{bmatrix} \cos(HA)\sin(90 - \delta) \\ \sin(HA)\sin(90 - \delta) \\ \cos(90 - \delta) \end{bmatrix} \tag{9}
$$

$$
\alpha = \arccos\left(\frac{TB \cdot TS}{|TB|\,|TS|}\right) \tag{10}
$$

We are able to make a significant simplification to the TS vector. Since the magnitude of the T vector is infinitesimal compared to the S vector, the angular difference between S and TS is negligible. Therefore, we can use the unit vector in the S direction.

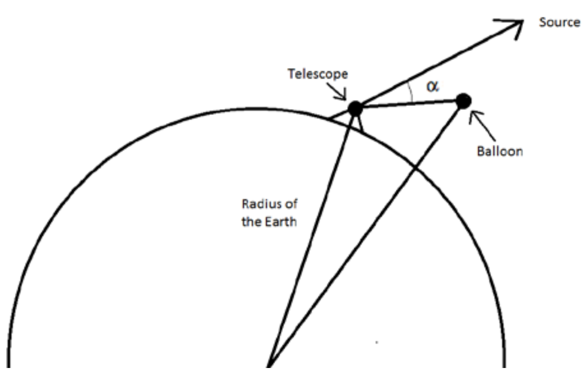

Figure 4: The angle problem.

C. Altitude and Azimuth

Another piece of information useful to astronomers is the direction of the balloon with respect to the telescope in terms of altitude and azimuth angles. This information is useful to astronomers in the process of locating and sending signals to the balloon. Altitude is the angle between the horizon and the balloon. Azimuth is the angle between the balloon and north, measured to the east.

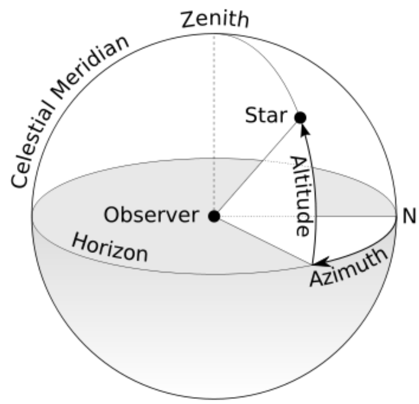

Figure 5: Altitude and azimuth.

With the use of some geometry and linear algebra, it can be shown that the altitude angle is defined by the following equation:

$$alt = \arccos\left(\frac{T \cdot TB}{|T|\,|TB|}\right) - 90^{\circ} \tag{11}$$

The derivation of azimuth is slightly more involved. In order to calculate azimuth, we must consider the latitude and longitude of the telescope, balloon, and an imaginary point P which shares the longitude of the telescope and the latitude of the balloon.

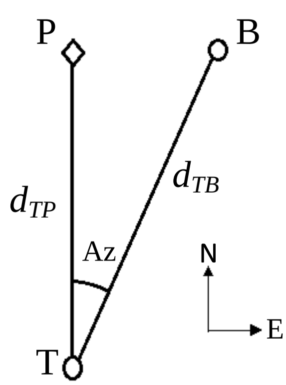

Figure 6: Azimuth. This image illustrates our method for calculating azimuth

First, we must find the length of two arcs between two points on a sphere in terms of their angular distance. These are the arcs between T and B and the arc between T and P. To find their lengths, we simply multiply the radius of the earth and their angular separation:

$$\cos\left(\Psi_{TP}\right) = \frac{T \cdot P}{|T|\,|P|} \tag{12}$$

$$\cos\left(\Psi_{TB}\right) = \frac{T \cdot B}{|T|\,|B|} \tag{13}$$

$$d_{TP} = R_E \Psi_{TP} \tag{14}$$

$$d_{TB} = R_E \Psi_{TB} \tag{15}$$

Since the PTB triangle is a right triangle, the azimuth angle will be given by:

$$az = \arccos\left(\frac{d_{TP}}{d_{TB}}\right) = \arccos\left(\frac{\Psi_{TP}}{\Psi_{TB}}\right) \tag{16}$$

This number can be adjusted by adding or subtracting multiples of 90 degrees to find true angle east of north.

VII. Future Steps

As we consider the future of this project, there are multiple areas where improvements could be made. We are currently developing new software tools which will allow us to perform the calculations from section VI in real-time. In order to launch the balloon towards the cone region, we would need accurate and real-time flight predictions. Also, some of our fellow researchers at Trevecca are developing a valve and ballast system to install on our balloons which will allow for more control over the flight path. Our research is still in the theoretical stage, but if these concepts prove to work as well in practice as they do in theory, they could revolutionize modern methods of radio astronomy.

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