Terminal Velocity of High-Altitude Balloon Payloads: Experiment Versus Theory

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The terminal velocity of a high-altitude balloon payload descending under a parachute can be calculated using the Prandtl expression for the drag force and knowing the force of gravity (weight) on the payload. A simple model of terminal velocity versus altitude has been developed, accounting for the changing density of the atmosphere during descent. This model will be compared to the actual terminal velocity of payloads launched by the University of Minnesota, Morris and ConHAB (Concordia College) balloon groups. We will also compare results between our two different parachute designs. The model and flight data will be used to develop an undergraduate laboratory activity illustrating differences between experimental real-world data and theoretically modeled data.

I. Introduction

THE increasing use of high-altitude ballooning has given students in both college and high school courses a unique opportunity to build and fly small payloads to the edge of space at very little cost. The payloads can be as simple as pointing a camera at an object and watching the effects of decreasing air pressure on it, or as complex as sampling for pollution at set altitudes along the flight path. We have developed an activity that uses several quantities that ballooning groups commonly record regardless of what other payloads they may be flying.

A typical flight involves a large latex weather balloon, about six feet in diameter, filled with helium. Underneath the balloon, a parachute is tied by string to a number of payload boxes comprising the stack. Due to Federal Aviation Administration rules, the total weight of the stack must be less than 12 pounds to avoid the need for a waiver from the FAA. A Concordia College (ConHAB) stack usually weighs around 9-10 pounds, and will reach 27,000 – 30,000 meters above sea level before the balloon bursts. At that point, the parachute takes over to slow the payload's descent back to the ground.

The flight profile to an altitude of 27,000+ meters sends the balloon through two distinct layers of the atmosphere and the boundary between them. From ground level to around 10,000 meters, the balloon is in the troposphere. The pressure and temperature both decrease with altitude in this layer, with temperatures possibly dropping to -60 ºC. At the top of the troposphere, the balloon enters a boundary layer called the tropopause, which is characterized by constant temperature for several thousand feet. After exiting the tropopause, the balloon enters the stratosphere, where temperature climbs due to the absorption of solar ultraviolet radiation by ozone at that altitude. Throughout the entire flight, pressure drops exponentially, to a minimum of around 6 mb or 1500 Pa. The temperature and pressure for a flight are plotted in Fig (1).

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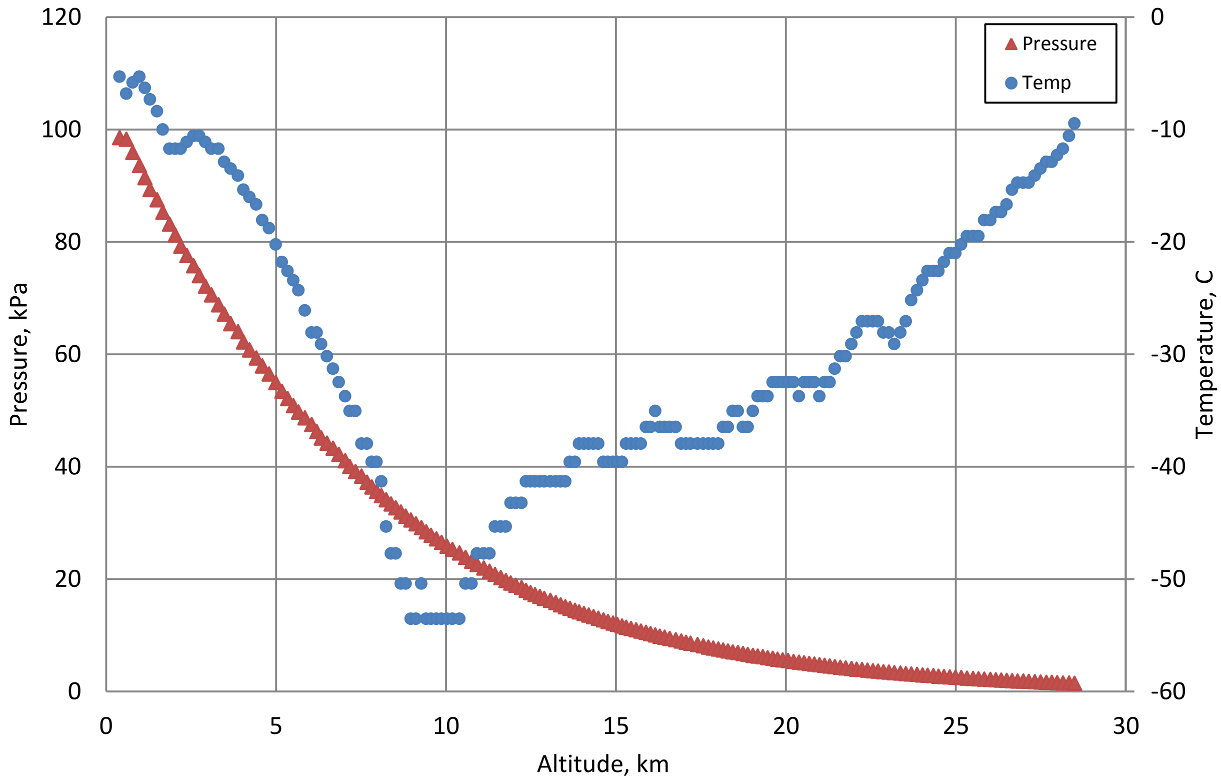

Figure 1. Pressure and temperature as a function of altitude. *From ConHAB's March 26, 2011 flight. Note the constant temperature near 10,000 meters when the payload was in the tropopause.*

II. The Model

Many undergraduate texts introduce the Prandtl expression[1] for the force of drag, *D*, that an object moving through air experiences:

$$D = \frac{1}{2} C_d \rho A v^2 \tag{1}$$

where C_d is the coefficient of drag, ρ is the density of air, A is the cross-sectional area of the object, and v is its velocity. The drag provides an upward force on a balloon payload descending under its parachute. The payload's weight provides the downward force. The payload and its chute will accelerate downward after the balloon bursts, and the whole package will reach its terminal velocity when the upward drag force grows large enough to equal the downward weight:

$$\frac{1}{2} C_d \rho A v^2 = mg. \tag{2}$$

This expression can be solved for v, giving the terminal velocity v_t,

$$v_t = \sqrt{\frac{2mg}{C_d \rho A}} \tag{3}$$

The drag coefficient and cross-sectional area for the parachute can be measured or taken from the parachute's manufacturer. A parachute's cross-sectional area can be defined in a number of ways. A common method, and the one used in our calculations, is the total area of the fabric of the chute as if it is laid out flat on the ground. Using this definition of A, most parachutes have a C_d in the range of 0.5 - 0.8.[2] The mass of the total payload and chute can be measured once the payload is recovered after the flight (this mass is always known before launch since one must know the mass of the stack in order to inflate the balloon to the proper amount of lift, but it is common for some random mass of the nozzle and bits of the balloon to come back to Earth. In addition, the amount of tape the students use to seal the payload boxes can vary widely). It should be noted that we are ignoring the drag due to the payload boxes, and only considering the effects of the parachute's drag, as that will be the dominant influence.

The air density is another matter though. It is hard to measure the density of the surrounding air as the payload is continuously either ascending or descending, but it is relatively easy to measure the air pressure and temperature at a given moment. The Ideal Gas Law can be stated in terms of the molar mass of air m and the individual gas constant R_{gas}

$$PV = mR_{gas}T \tag{4}$$

where P is the absolute pressure in Pascals, V is the volume in cubic meters, and T is the absolute temperature in Kelvin. This equation can be solved for m/V, which is simply the density of the atmosphere in terms of P and T,

$$\frac{m}{V} = \rho = \frac{P}{R_{gas}T} \tag{5}$$

The gas constant for air is 286.9 J kg^{-1} K^{-1}. A density profile for the descent of the ConHAB payloads as calculated by the pressure and temperature during the flight can be seen in Fig. (2).

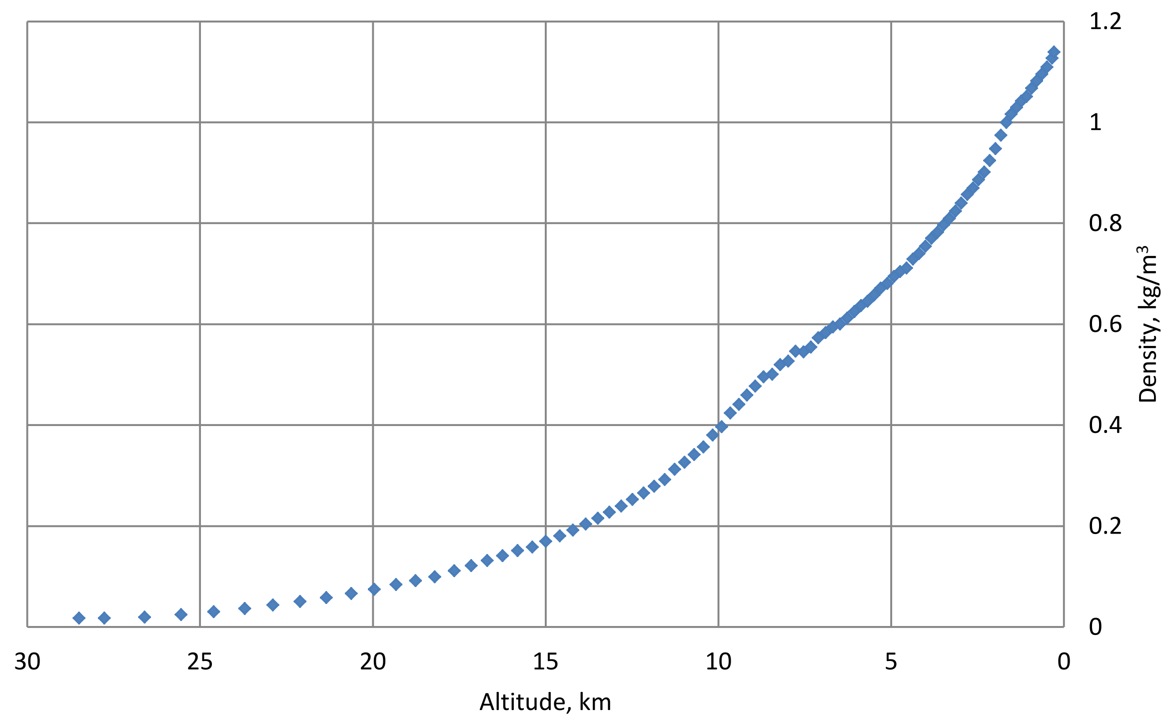

Figure 2. **Density vs. altitude.** *Calculated from the pressure and temperature measurements during the descent of the ConHAB March 26, 2011 flight.*

The density as shown in Eq. (5) is then substituted into Eq. (3) to determine the theoretical terminal velocity at any given altitude:

$$v_t = \sqrt{\frac{2mgR_{gas}T}{PC_dA}}. \tag{6}$$

III. Real-world Data

Now the calculated terminal velocity can be plotted against altitude and compared to the actual recorded velocity of the stack on descent as we have done for the University of Minnesota, Morris flight of March 26, 2011 seen in Fig. (3).

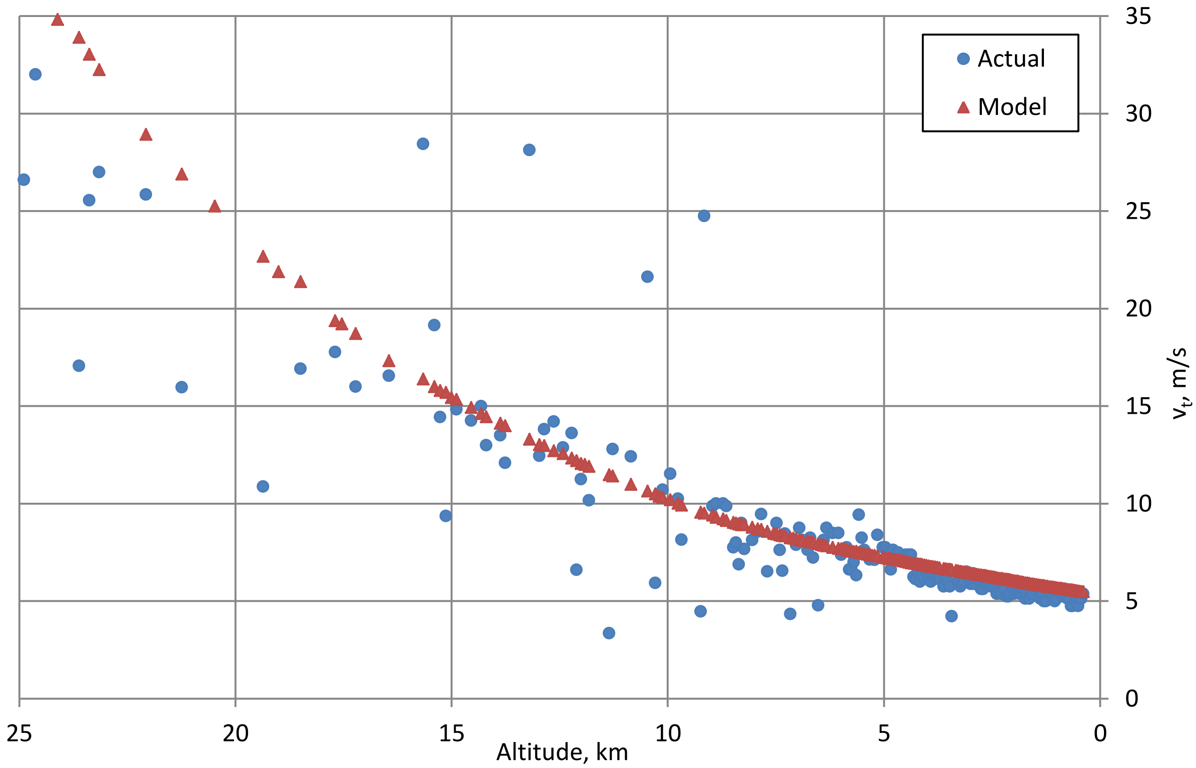

Figure 3. Actual v and modeled v_t as a function of altitude. *The actual data were recorded by the UMM StratoSAT command module on March 26, 2011.*

The UMM velocities were calculated from the GPS altitude and time data recorded by the StratoSAT command module. It is unknown at this time why the recorded data show such a large scatter initially. The modeled terminal velocity used pressure and temperature measurements from the StratoSAT command module. The diameter of the parachute was 72 inches, corresponding to an area of 2.62 m^2, and C_d was chosen to be 0.7. The combined mass of the payload and parachute was 3.53 kg. Despite the initial scatter of the actual data, by 15 km there is a nice correlation of the actual and model velocities.

The Concordia velocities were calculated from the GPS data in the APRS log file that were recorded by the tracking computer during the March 26 flight. The temperature data were recorded using a HOBO data logger, and the pressure was recorded with a pressure sensor and BASIC Stamp microcontroller built and programmed by a group of Concordia physics students during an experimental lab course. The recovered weight of the payload, parachute, and balloon remnants was 3.86 kg.

In reality, parachute aerodynamics is a very complicated subject, far beyond the simple model presented here. Many of the parameters of parachute design are determined experimentally. The UMM group uses a conventional hemispheric design, but ConHAB uses a cupped parabolic design. The parabolic design features triangular panels that extend downward from a parabolic canopy, significantly complicating any attempts at measuring the area. The primary advantages of the parabolic type over hemispheric type is parabolic has only four shroud lines (less tangling before launch) and better stability during descent. Using the model developed above, we can adjust the area and drag coefficient to fit the data, as seen in Fig. (4). The fit shown uses an area of 4.1 m^2 and a C_d of 0.6. The choice of C_d for this fitting was arbitrary (but still within the range of 0.5 – 0.8), so it is really only possible to determine the product of A and C_d.

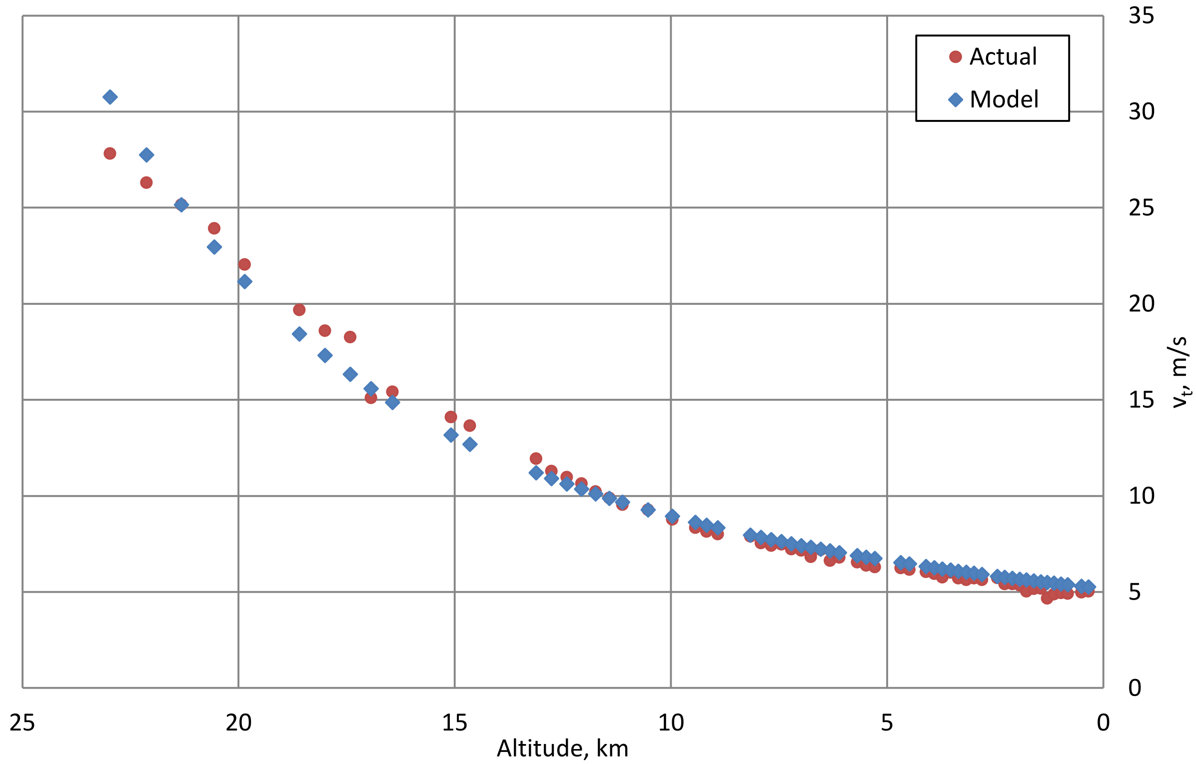

Figure 4. **Actual v and modeled v_t as a function of altitude.** *The model parameters were adjusted to fit the actual data from the ConHAB March 26, 2011 flight.*

IV. Conclusion

The comparison of the actual velocities with the modeled terminal velocities indicate that the parachute and payload fall at the terminal velocity throughout almost all of the descent. The terminal velocity is reached soon after burst, and the magnitude of the terminal velocity decreases as the parachute descends through the stratosphere into the lower, denser troposphere and down to the surface.

It is also interesting to note that both groups use parachutes with different designs, and the ConHAB payload was half a kilogram heavier, but both payloads fell at nearly the same rate through the last few thousand meters of the final descent. This comparison is a good illustration that different types of parachutes can show similar velocity profiles despite having different payload weights.

These data sets can be given to students in a laboratory setting, and they can calculate the theoretical descent rates and compare them to the descent rates of actual flights. It can be tailored in any way needed, from simply providing the numbers to having more advanced students develop the above model themselves. Discussions can be had about the differences between the theory and the real world, including things like drag due to the payload boxes, or how humidity and the increasing temperature of the stratosphere might affect air density. In the end, this is a simple equilibrium problem where the sum of the forces on the payload is equal to zero, suitable for an introductory lab activity.

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