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A Low-Cost Attitude Determination System using Multiple Sensors for High-Altitude Balloon Flights

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Presentation Overview

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- Background
- Attitude Determination
  - Markley's Singular Value Decomposition (SVD) Solution to Wahba's Problem
  - Sunlight Vector from Solar Cells
  - Acceleration Vector from Accelerometer
- Experiment
- Experimental Results
  - Angular Velocity Vector from Rate Gyroscope
  - Angular Velocity Vector from Estimated Attitude
  - Results & Discussion
- Future Developments
- Acknowledgments

Background

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- Attitude – the orientation of a body
- Attitude control used in dynamic systems:
  - Satellites
  - Aircrafts
  - Robotics
- Accurate attitude determination:
  - Low-cost sensors
  - High-altitude balloon payloads

- Growing interest:
  - Scientific and educational experiments at high altitudes:
    - Earth-related observations – geological and atmospheric
    - 60,000 to 120,000 ft.
  - Economical alternative to launch services
  - Project Loon, by Google [1]:
    - Provide internet to rural and remote areas

Attitude Determination

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- Direction Cosine Matrix (DCM):

$$\boldsymbol{r}^A = T_B^A \boldsymbol{r}^B$$

- Wahba's problem [2]:
  - Two-vector attitude determination
  - Sunlight and acceleration vector
  - Markley's SVD solution to Wahba's problem [3]
- Low-cost sensors:
  - Solar cells
  - 3-axis accelerometer

Wahba's Problem

$$\min_{T_B^A} J = \frac{1}{2}\sum_{n=1}^{N} a_n \|\boldsymbol{r}_n^A - T_B^A \boldsymbol{r}_n^B\|^2$$

where

$$N \geq 2$$

Markley's SVD Solution

$$B = \sum_{n=1}^{N} a_n \boldsymbol{r}_n^A (\boldsymbol{r}_n^B)^T$$

- Two-vector attitude determination:

$$B = \left(a_S \boldsymbol{r}_S^A (\boldsymbol{r}_S^B)^T\right) + \left(a_A \boldsymbol{r}_a^A (\boldsymbol{r}_a^B)^T\right)$$

- Singular Value Decomposition:

$$U \Sigma V^T = B$$

Markley's SVD Solution

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- DCM from SVD solution:

$$T_B^A = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & |U||V| \end{bmatrix} V^T$$

Inertial Frame Vectors

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- Acceleration vector:

$$r_a^A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- Sunlight vector:

$$r_S^A = \begin{bmatrix} 0.4449 \\ -0.7122 \\ -0.5430 \end{bmatrix}$$

Sunlight Vector from Solar Cells

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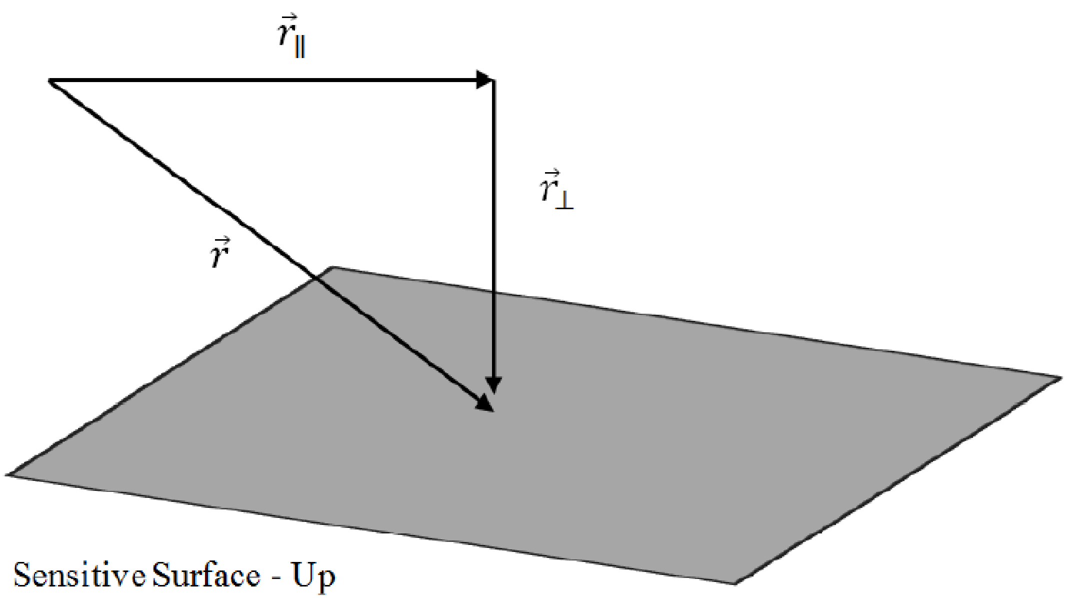
- Assumptions:
  - Flat device
  - Only one sensitive side
  - Only measures positive component of light information
  - Only measures normal component of light information



Solar Cells Used

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- PowerFilm Solar Inc. donated 26 of their SP3-37 solar cells:

[4]

Signal Conditioning

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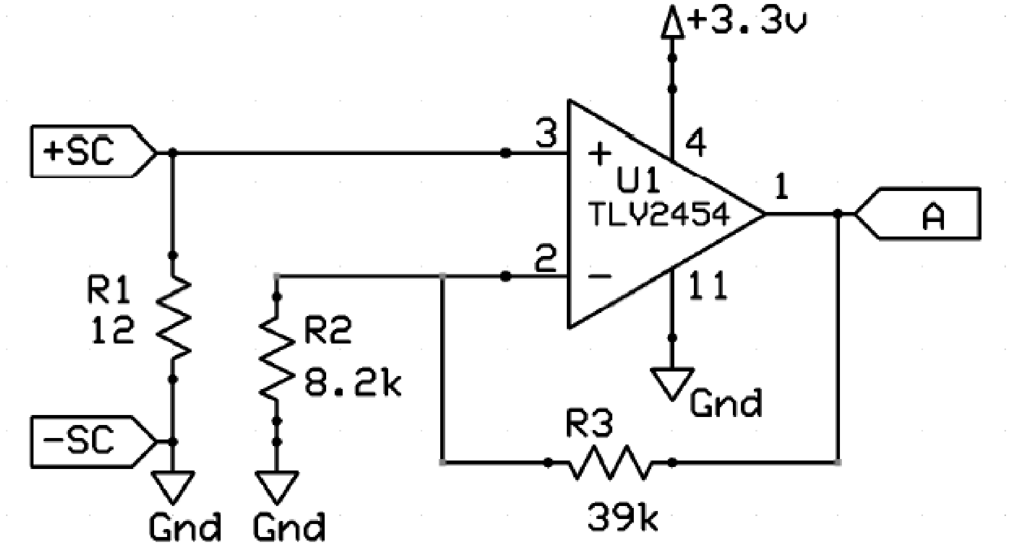
- TLV2454 rail-to-rail linear operational amplifiers (OPAMPs)
- Non-inverting configuration:



Sun Sensor Implementation

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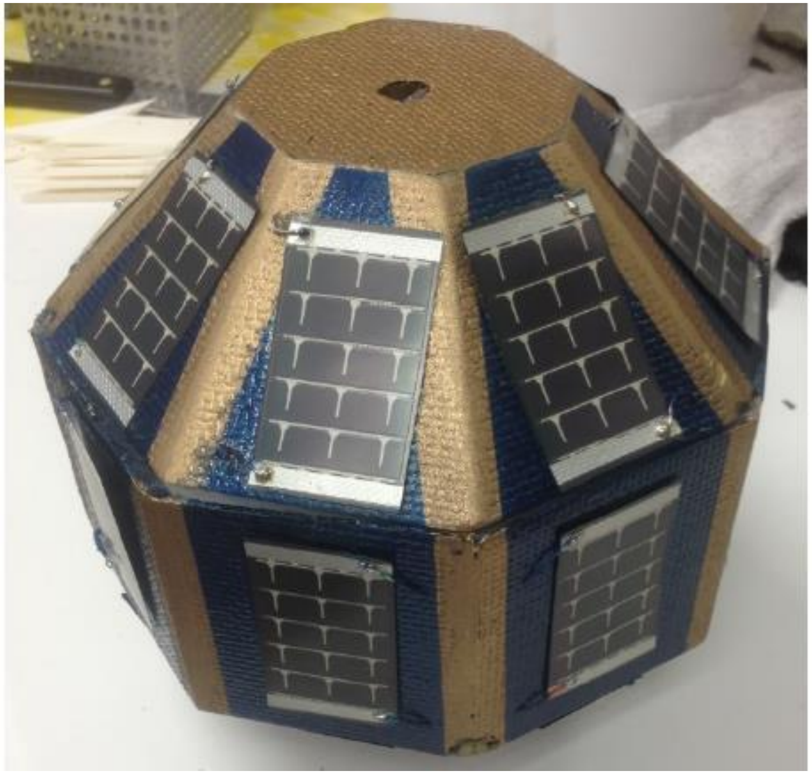

Sun Sensor Implementation

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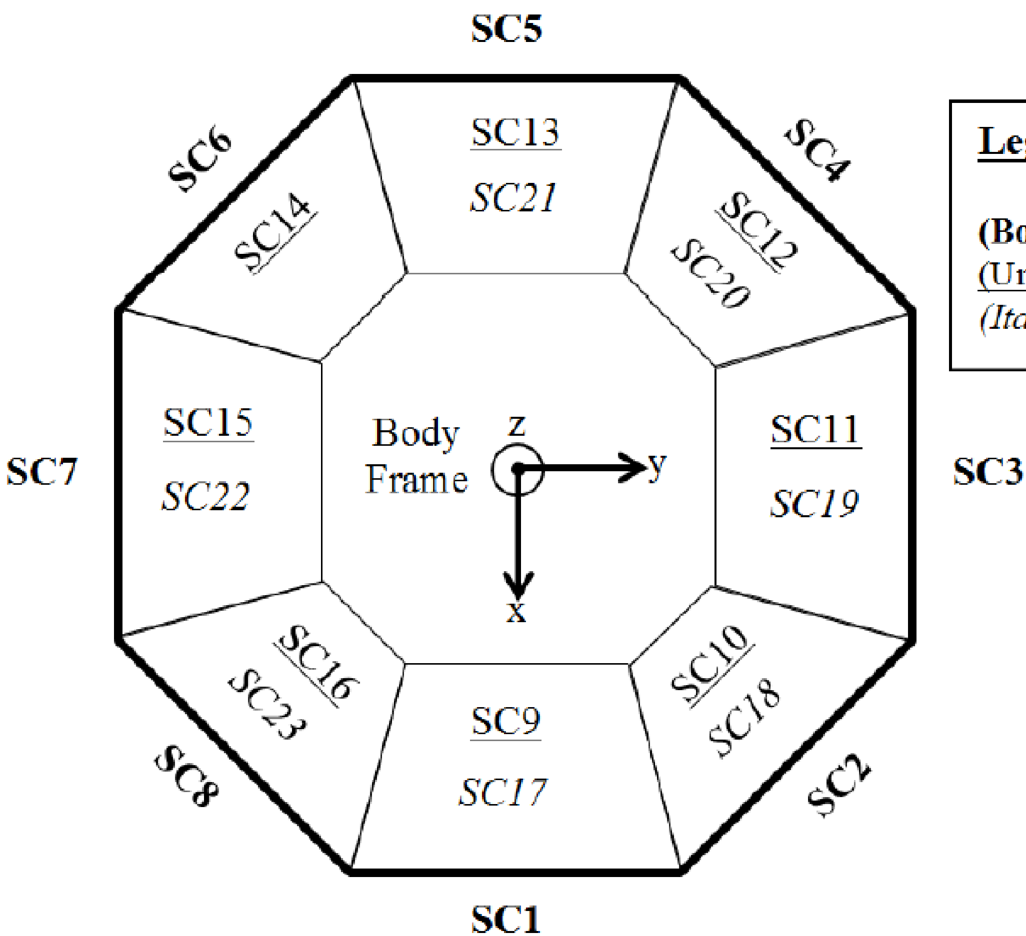

Legend:

(Bold) SC: Middle Layer Solar Cells
(Underlined) SC: Top Layer Solar Cells
(Italics) SC: Bottom Layer Solar Cells

Rotation Matrices

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$$Rot_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

$$Rot_Y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

$$Rot_Z(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Middle Layer Analysis

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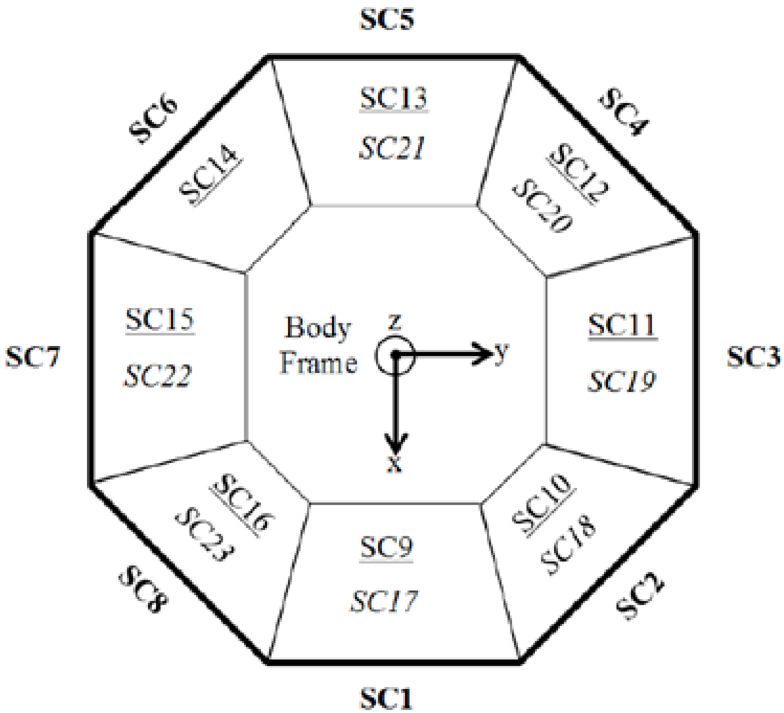

$$A_1 = [\ 1 \quad 0 \quad 0\ ](I)^T \boldsymbol{r}_S^B$$

$$A_2 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_3 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(\pi/2))^T \boldsymbol{r}_S^B$$

$$A_4 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(3\pi/4))^T \boldsymbol{r}_S^B$$

$$A_5 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(\pi))^T \boldsymbol{r}_S^B$$

$$A_6 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(5\pi/4))^T \boldsymbol{r}_S^B$$

$$A_7 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(3\pi/2))^T \boldsymbol{r}_S^B$$

$$A_8 = [\ 1 \quad 0 \quad 0\ ](Rot_Z(7\pi/4))^T \boldsymbol{r}_S^B$$

Top Layer Analysis

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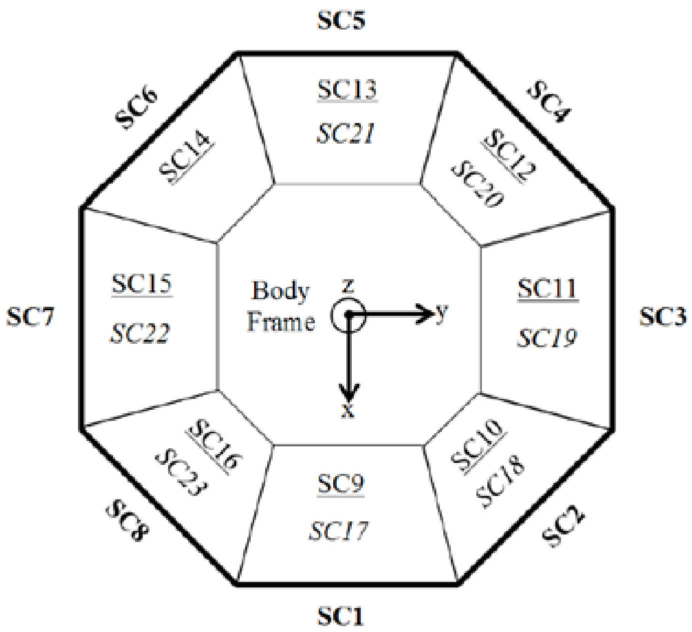

$$A_9 = [\ 1 \quad 0 \quad 0\ ](Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{10} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(\pi/4)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{11} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(\pi/2)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{12} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(3\pi/4)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{13} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(\pi)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{14} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(5\pi/4)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{15} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(3\pi/2)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{16} = [\ 1 \quad 0 \quad 0\ ](Rot_Z(7\pi/4)Rot_Y(-\pi/4))^T \boldsymbol{r}_S^B$$

Bottom Layer Analysis

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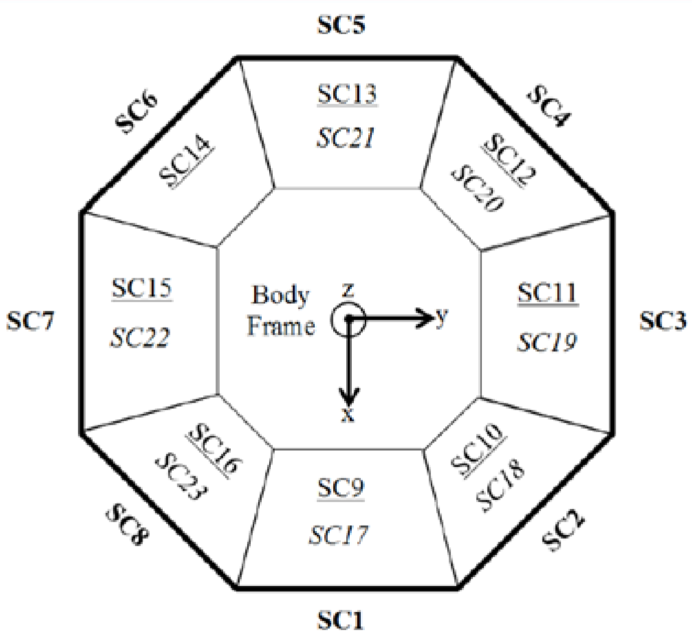

$$A_{17} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{18} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Z(\pi/4) Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{19} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Z(\pi/2) Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{20} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Z(3\pi/4) Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{21} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Z(\pi) Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{22} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Z(3\pi/2) Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

$$A_{23} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (Rot_Z(7\pi/4) Rot_Y(\pi/4))^T \boldsymbol{r}_S^B$$

Body Frame Sunlight Vector

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_{14} \\ A_{15} \\ A_{16} \\ A_{17} \\ A_{18} \\ A_{19} \\ A_{20} \\ A_{21} \\ A_{22} \\ A_{23} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -1 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & -1 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} r_S^B$$

- A9 through A13 missing due to wiring issues
- If Ai = 0; then, the corresponding equation must be removed

Acceleration Vector from Accelerometer

- Low-cost MEMs accelerometer used
- Assuming a linear input/output relationship:

$$\boldsymbol{V}_a = K\left(\ddot{\boldsymbol{r}}_a^B + \boldsymbol{g}^B\right) + \boldsymbol{c}$$

Accelerometer Calibration

$$\begin{bmatrix} \boldsymbol{V}_a^{x_g} - \boldsymbol{c} \\ \boldsymbol{V}_a^{y_g} - \boldsymbol{c} \\ \boldsymbol{V}_a^{z_g} - \boldsymbol{c} \end{bmatrix} = \begin{bmatrix} 9.81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.81 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9.81 & 0 & 0 \\ 0 & 9.81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9.81 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.81 & 0 \\ 0 & 0 & 9.81 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.81 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.81 \end{bmatrix} \begin{bmatrix} K_{11} \\ K_{12} \\ K_{13} \\ K_{21} \\ K_{22} \\ K_{23} \\ K_{31} \\ K_{32} \\ K_{33} \end{bmatrix}$$

Body Frame Acceleration Vector

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$$\ddot{\boldsymbol{r}}_a^B = \begin{bmatrix} 0.0234 & 0.0237 & 0 \\ -0.0151 & 0.0154 & 0 \\ -0.0004 & 0 & -0.0160 \end{bmatrix}^{-1} \left(\boldsymbol{V}_a - \begin{bmatrix} 1.4171 \\ 1.6419 \\ 1.8154 \end{bmatrix} \right)$$

Experiment

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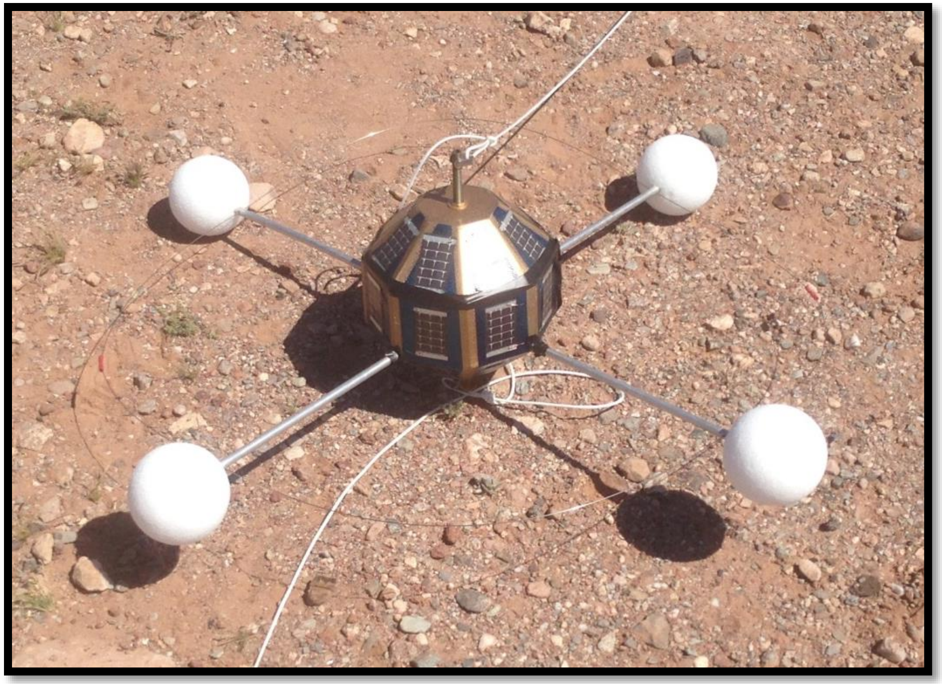
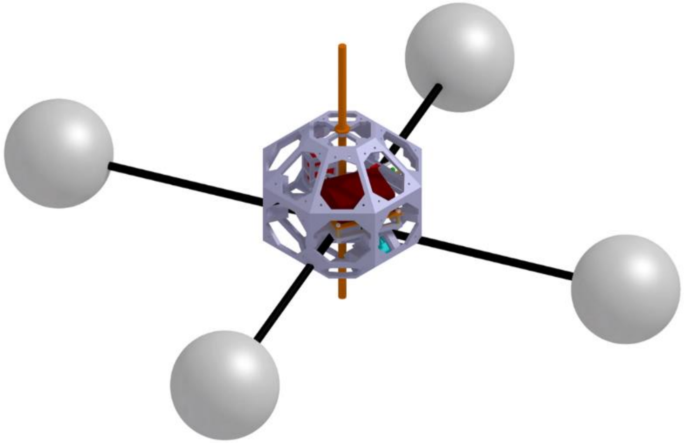
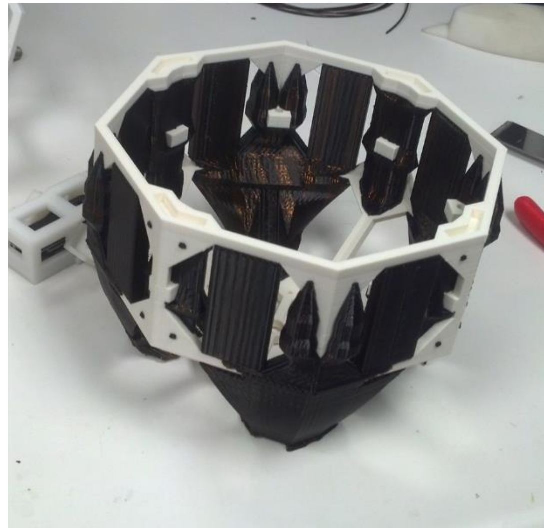
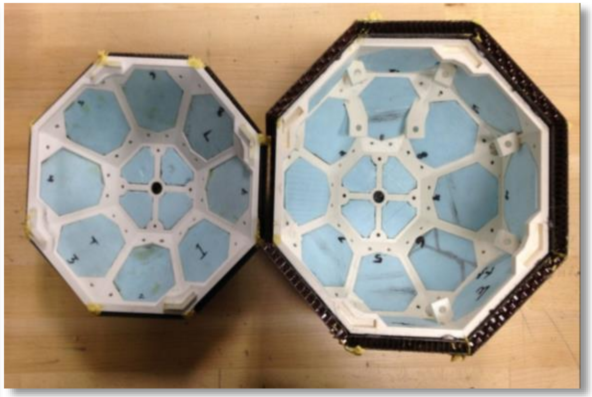
- Payload Overview:
  - 26-sided structure
  - Bi-directional carbon fiber
  - NOMEX honeycomb
  - Epoxy
  - Fiberglass reinforcement
  - 2/10" Styrofoam insulation
  - 3D printed internal structure

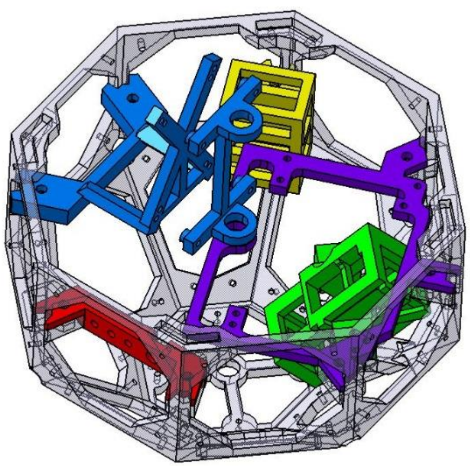


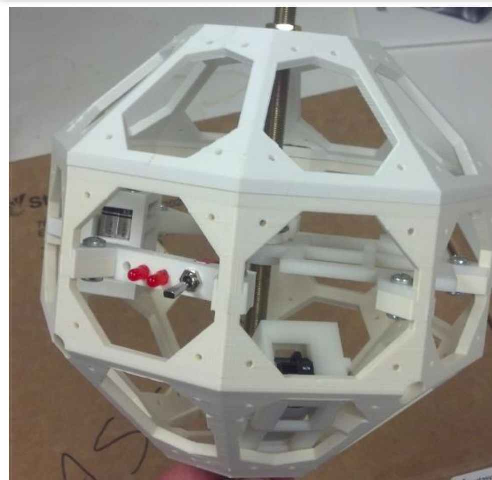
Payload

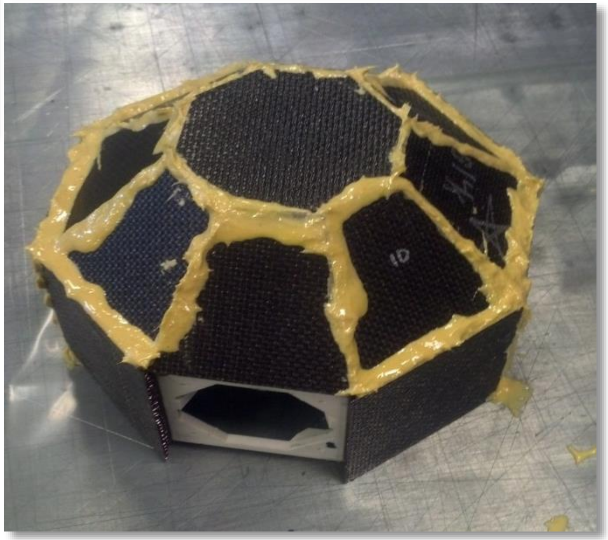












Payload

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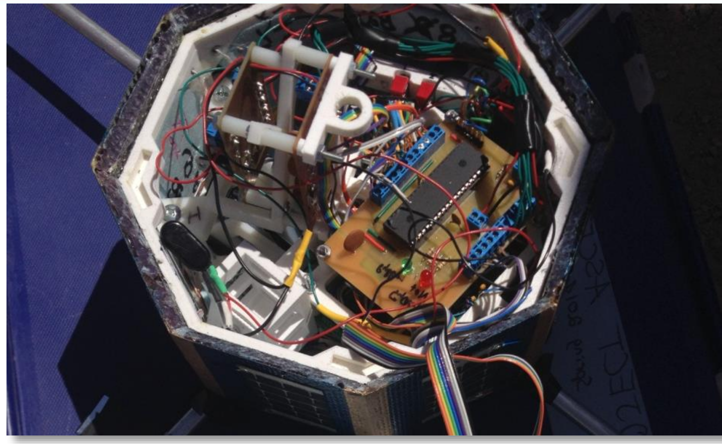

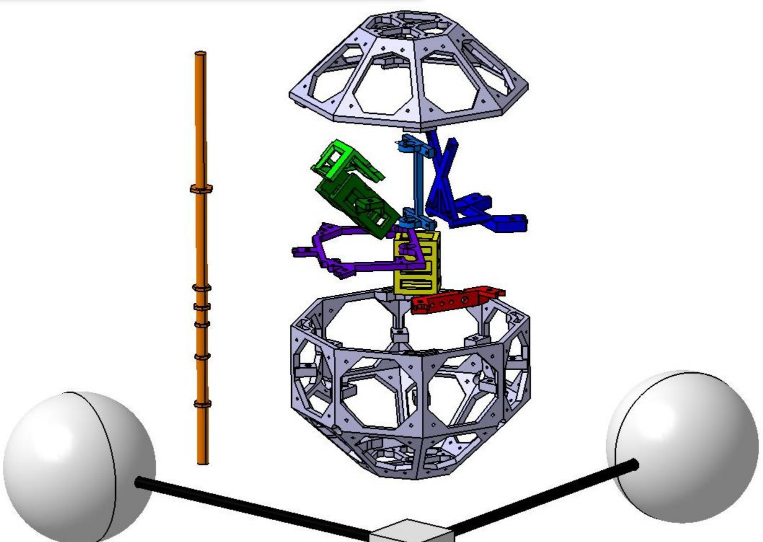

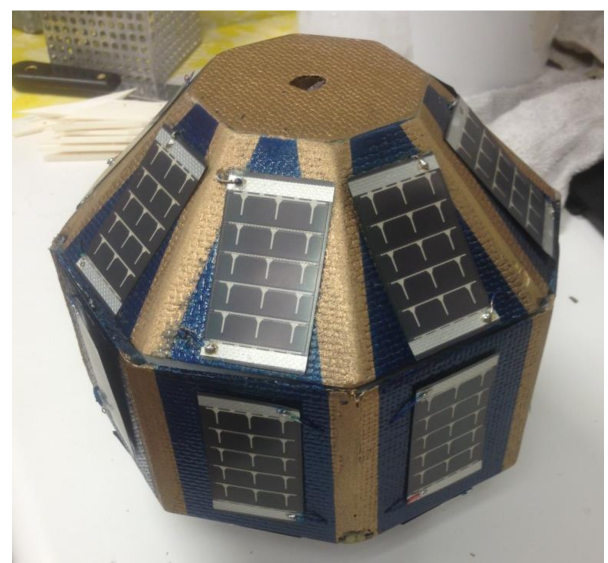

Flight

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Flight Details

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- ANSR (Arizona Near Space Research) flight for Arizona Space Grant ASCEND (Aerospace Scholarships to Challenge and Educate New Discovers) Project
- Flight Date: March 29th, 2014
- Burst Altitude: 73,794 ft.

Experimental Results

- No reference attitude is available to compare against
- Obtain angular velocity information from computed DCM (i.e., attitude), in the body frame
  - Compare against angular velocity information measured in the body frame during flight using rate gyroscope

Angular Velocity Vector from Rate Gyroscope

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- Low-cost MEMs rate gyroscope used
- Assuming a linear input/output relationship:

$$\boldsymbol{V}_{\omega} = S\boldsymbol{\omega}^{B} + \boldsymbol{b}$$

Rate Gyroscope Calibration

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$$\begin{bmatrix} \mathbf{V}_a^{x_{rot}} - \boldsymbol{b} \\ \mathbf{V}_a^{y_{rot}} - \boldsymbol{b} \\ \mathbf{V}_a^{z_{rot}} - \boldsymbol{b} \end{bmatrix} = \begin{bmatrix} \omega_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \omega_x & 0 & 0 \\ 0 & \omega_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_y & 0 \\ 0 & 0 & \omega_z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_z \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{12} \\ S_{13} \\ S_{21} \\ S_{22} \\ S_{23} \\ S_{31} \\ S_{32} \\ S_{33} \end{bmatrix}$$

Body Frame Angular Velocity Vector from Rate Gyroscope

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$$\boldsymbol{\omega}^B = \begin{bmatrix} 0.0032 & 0.0026 & -0.1941 \\ -0.1363 & 0.1327 & 0.0025 \\ 0.1399 & 0.1394 & 0.0056 \end{bmatrix}^{-1} \left(\boldsymbol{V}_\omega - \begin{bmatrix} 1.4865 \\ 1.4892 \\ 1.4844 \end{bmatrix} \right)$$

Angular Velocity Vector from Estimated Attitude

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$$\dot{T}_B^A = T_B^A \begin{bmatrix} 0 & -\omega_z^B & \omega_y^B \\ \omega_z^B & 0 & -\omega_x^B \\ -\omega_y^B & \omega_x^B & 0 \end{bmatrix}$$

- Central difference method
- Ideal low-pass filter:
  - Fourier transform of signal
  - Break frequencies (x, y, and z): 2.3, 2.0, and 0.8 (rad/s)
  - Inverse Fourier transform of result

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Results

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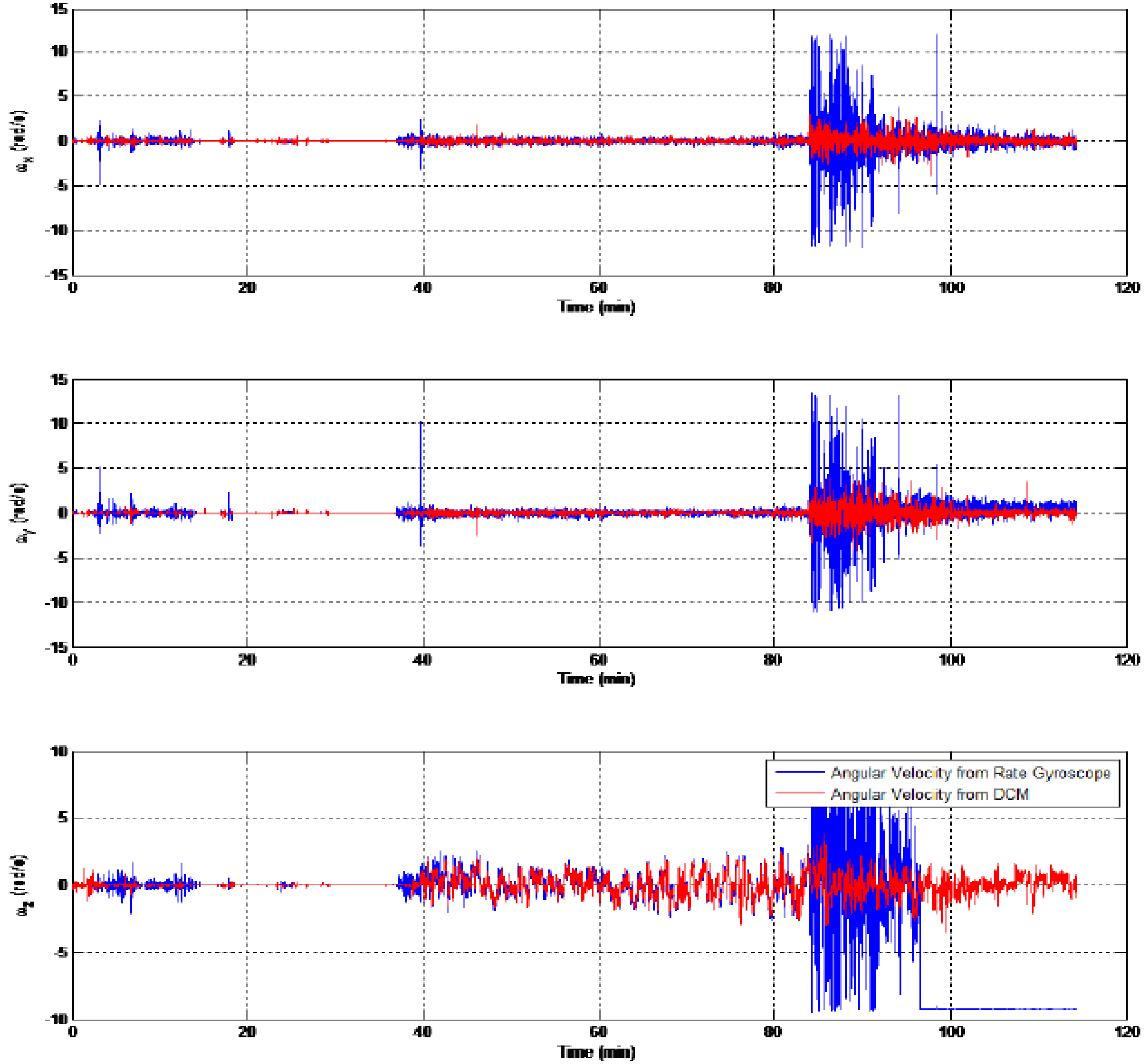
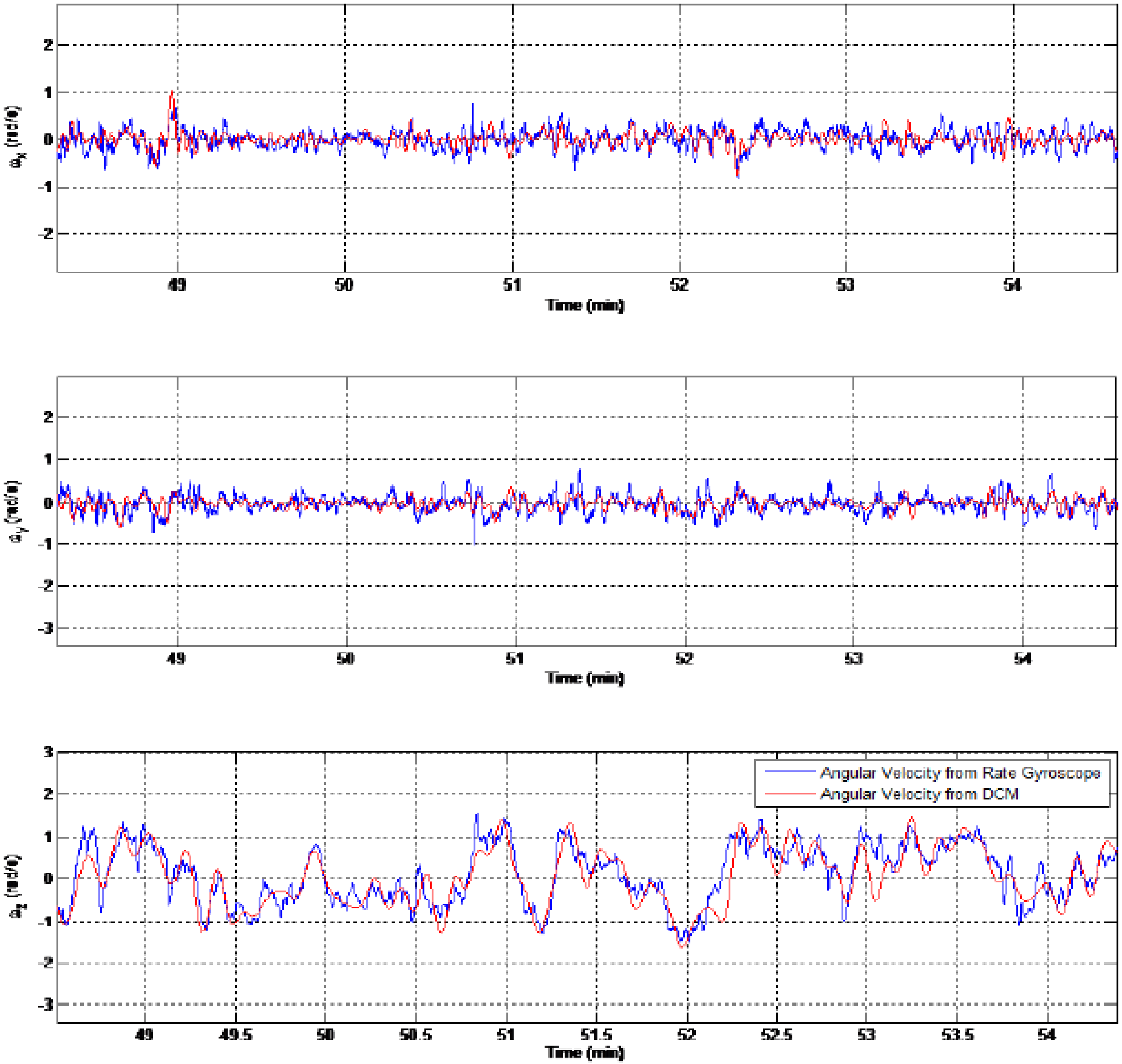


Results


2
1
0
-1
-2
ωx (rad/s)
49
50
51
52
53
54
Time (min)
2
1
0
-1
-2
-3
ωy (rad/s)
49
50
51
52
53
54
Time (min)
3
2
1
0
-1
-2
-3
ωz (rad/s)
Angular Velocity from Rate Gyroscope
Angular Velocity from DCM
49
49.5
50
50.5
51
51.5
52
52.5
53
53.5
54
Time (min)

Future Developments

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- More than two independent vector measurements
- Two-vector attitude determination:
  - Magnetic field vector
  - Sunlight vector
- Compare against another attitude:
  - Roll, pitch, and yaw using image/video processing

Acknowledgements

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Aeronautical University

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  - Dallas Hodge
  - Austin Leonard
  - Patrick Deskin
  - Loren Williams
  - John Cybulski
  - Brigette Cochran

Questions?

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References

[1] http://www.google.com/loon/, \Google Loon," 2014.

[2] Wahba, G., \A least squares estimate of satellite attitude," SIAM review, Vol. 7, No. 3, 1965, pp. 409{409.

[3] Markley, F. L., \Attitude determination using vector observations and the singular value decomposition," The Journal of the Astronautical Sciences, Vol. 36, No. 3, 1988, pp. 245{258.

[4] *SP3-37 Solar Cells*. Digital image. *PowerFilm Solar Inc*. N.p., n.d. Web.