

Self-assembly of aperiodic tilings

Ilya Galanov

LIPN - CNRS & Univ. Paris 13

galanov@lipn.univ-paris13.fr

Aperiodic tilings serve as a mathematical model for quasicrystals - crystals that don't have any translational symmetry (Penrose tiling is one famous example). The objective is to grow such a tiling by adding tiles one by one using only the local information. The motivation is to mimic the growth of a real world quasicrystals. In this talk we propose a local growth algorithm for a particular class of aperiodic tilings namely *octagonal tilings of finite type*, that is, cut and project tilings in four-dimensional space which admit local rules [1].

In year 1992 Joshua Socolar published a paper about local self-assembly algorithm for Penrose tilings [2]. The following was proposed: In a case, when only one tile fits according to the *matching rules*, it should be added. Otherwise nothing is added except for the special cases when the decision on what tile to add is probabilistic, based on a constant α . Socolar proved that given an arbitrary radius R and the probability P arbitrary close to unity, one can choose α so that the algorithm will produce a correct pattern of Penrose tiling that covers the disk of radius R with probability greater than P .

We attempt to generalize the Socolar's algorithm to work not only with Penrose tilings but with a class of octagonal tilings of finite type and also exclude the probabilistic aspect. Simulations support the conjecture that given a tiling from the class one can cover an arbitrary proportion of the plane (but not all of it) by choosing suitable constants in the algorithm and the starting seed to be big enough (see *Figure 1*). We will illustrate the conjecture on two examples (namely the Penrose tiling and the Golden-Octagonal tiling).

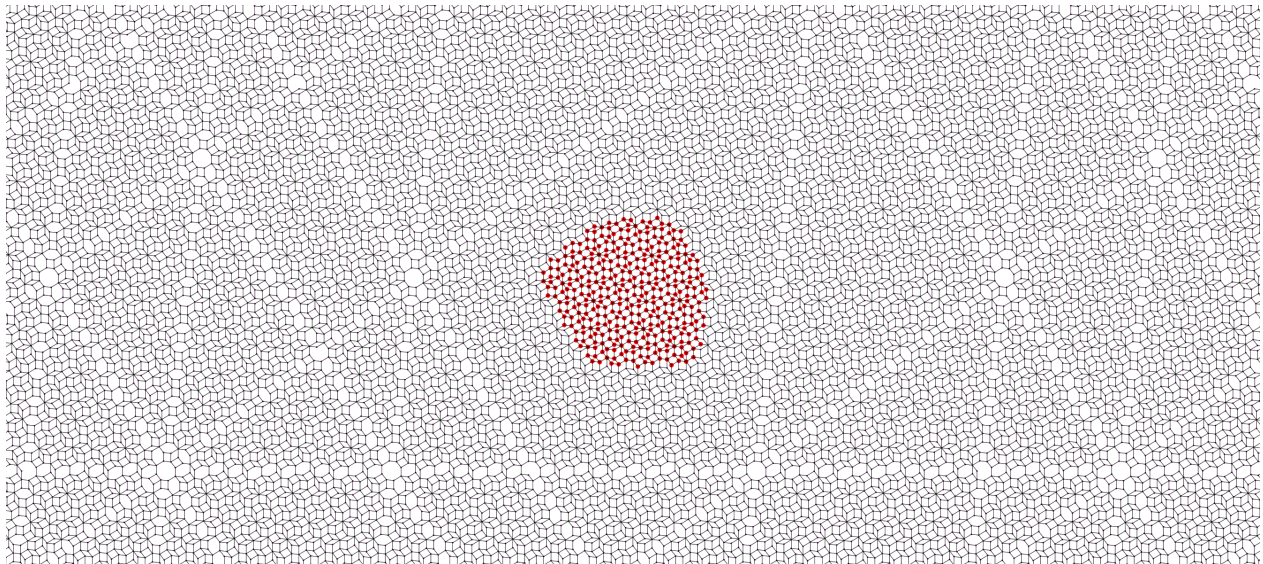


Figure 1. Golden-Octagonal tiling. The seed is marked with red. The whole plane is to be covered except for the stripes of so called *flips*. Choosing a bigger seed will result in some of the empty stripes to be filled.

1. N. Bedaride, Th. Fernique, Weak local rules for planar octagonal tilings, *Israel Journal of Mathematics*, The Hebrew University Magnes Press, 2017, 222 (1), pp.63 - 89.
2. J. E. S. Socolar, Growth rules for quasicrystals, *Quasicrystals: the State of the Art*, edited by D. P. DiVincenzo and P. J. Steinhardt (World Scientific Publishing Co., Singapore, 1991), pp. 213–238.