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By Dr. Louis G. Reifschneider

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Dr. Louis Reifschneider teaches product design and manufacturing technology related courses at Illinois State University with an emphasis in plastics processing. His research interests involve using computer-aided engineering software for product design as well as developing ways to process bio-materials with conventional plastics processing equipment. Dr. Reifschneider holds a B.S. in Mechanical Engineering from the University of Notre Dame (1983), and M.S. in Mechanical Engineering from the University of Minnesota (1984) and earned his Ph.D. in Mechanical Engineering from Ohio State University (1990.)

Teaching Kinematic Synthesis of Linkages Without Complex Mathematics

By Dr. Louis G. Reifschneider

Introduction

Courses in which the design of machine elements is taught typically address the design of four-bar linkages, cams, and gear trains. Instruction about the function of mechanisms begins with kinematic analysis where it is assumed that all principal dimensions of a mechanism are known, the interconnections of the links are defined, and the motion of the driver link is prescribed. The task is to determine the displacements, velocities, and accelerations of the various members in the mechanism and also the path followed by certain elements (Erdman, Sandor, & Kota, 2001). Kinematic synthesis, on the other hand, begins with a prescribed motion that must be achieved with an as yet unknown sized or shaped mechanism. It is the intent of the author to show by detailed examples how graphing the trends observed while performing the overlay technique for kinematic synthesis can be used to facilitate finding a solution to a common set of design problems that would otherwise require complex mathematics to solve.

At the undergraduate level, the topic of kinematic synthesis is often limited to examples of gear trains and cams because only algebra and trigonometry are needed to solve many synthesis problems of these types. However, synthesis of mechanical linkages that achieve a specific output motion with a specific input motion, called function generation, requires either tedious, iterative methods or the use of advanced mathematics such as nonlinear equation solvers and complex number methods (Erdman, Sandor, & Kota, 2001). The latter are typically beyond the scope of undergraduate technical curricula. To address the difficulties posed when teaching kinematic synthesis of linkages, the author has implemented a graphing-based technique that can facilitate the linkage design process for students and thus enable them to effectively design four-bar mechanisms with the aforementioned types of constraints. Such mechanisms are widely used in the design of material handling systems, jigs and fixtures, as well as robotic manipulators. Further, the graphing-based technique, when coupled with parametric computer-aided-drafting greatly facilitates the traditional iterative drawing technique used to solve such problems.

Some instructors of mechanism design already infuse mechanism synthesis in their courses (Balamuralikrishma & Mirman, 2003). It is hoped that the method outlined in this article with the detailed examples will foster more design synthesis activity within manufacturing technology programs. The author has successfully implemented this technique within an undergraduate mechanism design course in which students performed design synthesis of a linkage mechanism of their creation and then verified the design using the kinematic analysis techniques learned in the course. The designs were limited to four bar mechanisms but ranged in function from bottle openers, to robotic grippers, to can crushers, to scissor tables.

Background

The focus of this paper is on the design of four-bar linkages because they are the starting point when teaching kinematic analysis of mechanisms (Myszka, 2005; Erdman, Sandor, & Kota, 2001; Wilson & Sadler, 2003). The first step in mechanism design involves creating a conceptual design that will perform the specified function, usually a prescribed path of motion. The critical stage in the design process is determining the functional and mechanical dimensioning of the mechanisms that make up the machine, (Angeles, 2003). Kinematic synthesis is performed to determine the functional dimensions. The synthesis process determines the lengths of links and the distances between pivot points required to achieve the specified motion of the device.

Kinematic analysis and kinematic synthesis can be performed either graphically or with the aid of mathematical relationships. Table 1 summarizes the type of mathematics required to perform each task for the three basic types of mechanisms. Techniques that are used to perform kinematic analysis of linkages include:

1. Redrawing a linkage pattern while obeying joint constraints and link dimensions with either manual drafting or with the aid of computer-aided-drafting (CAD). This technique is also called the overlay method as the shape of the linkage is redrawn in a slightly modified position.
2. Automating the CAD drawing with parametric models (wire frame or solid models) of the linkage that impose geometric constraints and dimensional constraints to simulate the motion of the mechanism. Programs that perform this task include Working Model (2004), ADAMS (2004), Unigraphics NX (2004), Pro-Engineer (2004).
3. Finally, the mathematical relationships can be developed to represent the positions of the linkage elements as the input driver link is moved; this is done by solving trigonometric equations.

Synthesis of Linkages

The solution method used to solve a kinematic synthesis problem depends upon the type of problem being solved. The most basic types of kinematic synthesis of four-bar mechanisms are two-point and three-point position synthesis which involve using geometric bisection drawing techniques to locate the pivot points that allow a link in the mechanism to achieve two or three prescribed positions ((Myszka, 2005; Erdman, Sandor, & Kota, 2001; Wilson & Sadler, 2003). Sometimes, however, the pivot points are already defined and the design task is to determine the length of links that will allow the mechanism to achieve two prescribed angles or two positions of other links in the mechanism. This is called a function generation type problem and these are the focus of this article. For example, assume the offset crank-slider design shown in Figure 1 must place the slider at a horizontal offset of 2 inches when the crank angle is at 60 . In addition, this same mechanism must place the slider at a horizontal offset of 4 inches when the crank is at 10 . Observe in Figure 1 that if the crank maintains a length of 1 inch, the coupler must change length from 2.394 to 3.236 inches to achieve the two prescribed orientations of the crank angle and slider position. Therefore, the design shown in Figure 1 cannot achieve the prescribed motion. However, it is possible to find a combination of crank and coupler lengths that will satisfy both the "60 at 2 inches" and the "10 at 4 inches" constraints. They can be determined iteratively using an overlay technique as illustrated in Figure 2. The solid lines in the figure represent an overlay of the series of crank-coupler pairs that satisfy the "60 at 2 inches" constraint. The dashed lines in Figure 2 illustrate the overlay of crank-coupler pairs that satisfy the "10 at 4 inches" constraint. The solution sought with the overlay method can become difficult to discern as many lines and dimensions are drawn in the graphic window. However, by making a graph of the coupler lengths for each constraint as the crank length changes, as shown in Figure 3, the trend towards a solution becomes evident. The graph in Figure 3 illustrates there is a combination of crank-coupler lengths that will satisfy both constraints. The designer can use this information to render crank lengths closer to the optimal shown in the graph to better approximate a solution to the design problem.

It is important to note this method will yield only an approximate solution to the design problem. The same is true for any iterative solution method used to solve the nonlinear mathematics that governs this type of design. The nonlinear nature of kinematic synthesis will be shown later in this article. However, as in all iterative methods, the error in the approximation can be made so small as to be practically insignificant. In summary, there are several techniques that can be used to perform kinematic synthesis depending upon the type of synthesis problem:

1. Use geometric construction to locate pivot points for prescribed two-point and three-point synthesis of a single link as described in Myszka (2005) and Erdman, Sandor, & Kota (2003).
2. For function generation type problems, manually or with CAD, redraw the mechanism while varying the elements of the mechanism that are free to change to satisfy the desired motion. This technique is called the overlay method and is described in detail in this article and elsewhere in Erdman, Sandor, & Kota (2001).
3. Using parametric CAD, constrain as previously mentioned, but then allow the unknown element to freely change angle or position as the range of possible configurations are drawn for the other free link. This approach is essentially a faster overlay method as illustrated in Figure 2.
4. For simple linkage synthesis, for example a four-bar mechanism with two prescribed positions and constraints on pivot positions and angles, develop and solve the nonlinear trigonometric equations with proper solution techniques. Complex number methods (Erdman, Sandor, & Kota, 2001) and dot product methods (Wilson & Sadler, 2003) have been developed to express the relationship between the length of links and angles in order to solve for the mechanism geometry.
5. For more sophisticated linkage synthesis (more than four links and or more than two prescribed positions), employ commercial programs designed to determine the optimally sized linkage that generates motion through a prescribed path defined by a finite number of points. One program specifically designed to do this is LINCAGES 2000 (Ning, Erdman & Byers, 2002) and WATT Mechanism Design (Draijer & Kokkeler, 2002). Multibody kinematic analysis and optimization programs that can also perform this type

of analysis include the complement of MotionView (2004) and HyperStudy (2004).

Practical Examples

To motivate instructors to consider implementing a practical kinematic synthesis problem in their mechanism design class, two examples are discussed in detail. The first design addresses a need to open a furnace oven door with a hand crank. Due to ergonomic considerations, the designer would like a specific crank angle to close the door and another crank angle to fully open the door. A schematic of basic oven design is shown in Figure 4. A second design problem involves using a pneumatic cylinder to maintain a platform at two prescribed angles. The cylinder has a prescribed travel: a fully retracted length and a fully extended length. A schematic of this mechanism is shown in Figure 10. In addition to the overlay technique, this second design problem will be used to illustrate the type of nonlinear mathematics that result when trying to solve kinematic synthesis problems of linkages. It will also be used to show how graphing the nonlinear relationships within the mechanism can be used to determine solutions while using minimal CAD overlay rendering.

The first example involves the oven door opening linkage shown in Figure 4 which is an offset four-bar crank slider mechanism. The kinematic diagram of this mechanism is shown in Figure 5. In this design, the offset distance, L1 and the slide distance, L4 are specified. The slide distance is the vertical distance from the oven pivot to the coupler-slider joint when the door is fully closed. This represents the maximum downward travel of the door (the slider) in this problem. The design constraints are as follows:

1. The door should be closed when the crank angle is at 10^o and
2. The door should be opened 4 inches when the crank angle is at 30^o.

The design synthesis task is to find the combination of crank and coupler lengths that can best satisfy these two constraints. Because the mechanism is driven by the crank angle, it is best to randomly select various crank lengths and determine the associated coupler length. The same crank lengths will be used for each crank angle and the corresponding coupler lengths will be determined by a CAD program. The overlay technique used to solve this design problem is shown in Figures 6 and 7. In addition to the two constraints, two criteria are established to determine when an optimal solution is found:

1. If the percentage difference between the coupler lengths satisfying each condition is less than 0.1%, then a solution is found. This is a convergence criteria and is needed for any iterative solution problem, further,
2. The combination of the chosen crank and coupler must satisfy the most critical tolerance for the design.

Because the solution is found by iterating until a finite convergence criteria is met, the converged pair of crank-coupler values will only exactly satisfy one of the position constraints: either the “closed door” or the “open door”. Because the door functions to keep the heat inside the oven, as a practical matter for this problem the “closed door” condition must be satisfied. It is not acceptable to have the door “almost closed”.

A summary of the CAD data obtained from the overlay technique is given in Table 2. The converged solution is shown with a bolded font. A graph of the preliminary overlay data and then the final overlay data is shown in Figure 8. It should be pointed out in this case the 13.0, 13.3, and 13.5 inch crank values were not rendered with the CAD program until a graph was made of the other values shown in Figure 8A. The solution of 13.300 inches for the crank and the corresponding “closed door” coupler value of 16.399 inches are used to render the closed and the opened positions of the oven, shown in Figure 9. Note, although the door closes to 0.00, the open position is not 4.00 inches but 4.015. For this mechanism and the prescribed convergence criteria, an error of 0.015 inches results for the position of one of the two positional constraints. The 0.015 inch error could be made smaller if the convergence criterion is made smaller. It is up to the judgment of the designer whether this error is acceptable.

In a second example, the type of nonlinear mathematics that results when performing an analytical solution to a linkage synthesis problem is outlined. The problem involves sizing the length of the base and the length of the rocker link for a pneumatic tilt control mechanism shown in Figure 10. The mechanism is to be designed to achieve the following two positions:

1. When the cylinder is fully retracted to 8 inches, the incline angle should be 30^o,
2. When the cylinder is fully extended to 12 inches, the incline angle should be 60^o.

Because this problem involves determining the length of two sides of a triangle, the laws of cosines, the laws of sines, and the rules of triangles can be used to derive the analytical relationships needed to solve this problem.

Analytical Methods

The analytical relationships that govern the motion of this mechanism are derived to illustrate how nonlinear equations must be solved to determine a solution to this synthesis problem and by extension how such methods are needed to solve other four-bar mechanism problems. To begin, the relationship between the cylinder length and the length of the rocker and base is governed by the law of cosines:

$$\text{Cylinder\_Length}^2 = \text{Rocker}^2 + \text{Base}^2 - 2\times\text{Rocker}\times\text{Base}\times\cos(\text{Incline\_Angle}) \quad (1)$$

To illustrate how the mathematics required for kinematic analysis differs from that needed for kinematic synthesis, a typical kinematic analysis problem will be solved first. In this case, assume the cylinder shown in Figure 10 retracts to 10 inches; can the new incline angle be determined? Because all of the link lengths are known, the value of the new incline angle can be directly computed by the solving for Incline_Angle from Eq. 1:

$$\text{Incline\_Angle} = \cos^{-1}\left(\frac{\text{Rocker}^2 + \text{Base}^2 - \left(\text{Cylinder\_Length}^2\right)}{2 \times \text{Rocker} \times \text{Base}}\right) \quad (2)$$
$$= 45.6^{\circ}$$

This example of kinematic analysis is performed by making direct calculations of trigonometric formula. Kinematic synthesis, on the other hand, has a more complex set-up and solution. For the design problem posed, the following nomenclature will be used to identify the position constraints:

- Retracted constraint: Cylinder_short = 8 inches and Incline_short = 30^{o}, lower incline angle.
- Extended constraint: Cylinder_long = 12 inches and Incline_long = 60^{o}, higher incline angle.
- Rocker length denoted by the letter "a".
- Base length denoted by the letter "b".

The synthesis solution involves finding the values of "a" and "b" that satisfy the two prescribed positional constraints. The solution is found by applying the law of cosines, as before, and the law of sines as well as the rule that the sum of all angles in a triangle equals 180. First, apply the law of cosines for both constraints:

By subtracting Eq. 3 from Eq. 4, the product a x b can be isolated.

$$a \times b = \left(\frac{0.5 \times \left(\text{Cylinder\_long}^2 - \text{Cylinder\_short}^2\right)}{\cos(\text{Incline\_short}) - \cos(\text{Incline\_long})}\right) \quad (5)$$

After applying the law of sines and the sum of angles rule, another equation for the unknowns a and b results:

$$\sin^{-1}\left(\left(\frac{a}{\text{Cylinder\_long}}\right)\sin(\text{Incline\_long})\right) + \sin^{-1}\left(\left(\frac{b}{\text{Cylinder\_long}}\right)\sin(\text{Incline\_long})\right) = 180 - \text{Incline\_long} \quad (6)$$

Finally, by solving for "a" in Eq. 5 and substituting into Eq. 6, a nonlinear equation for "b" results shown in equation 7. The letters "X", "Y", and "Z" in Eq. 7 are constants that are functions of known link lengths and angles and are used to simplify the expression.

$$\sin^{-1}\left(\frac{X}{b}\right) + \sin^{-1}(b\,Y) = Z \quad (7)$$

The solution for "b" in Eq. 7 involves iterative solution methods which are typically beyond the scope of most undergraduate programs in technology or engineering. Consequently, the synthesis of a design for the simple positioning mechanism shown in Figure 10 appears beyond the scope of an undergraduate design course. However, the overlay technique can be used by employing a CAD program and, as before, the overlay results can be plotted to reveal where a solution exists.

The overlay method for this problem is illustrated in Figure 11 for the retracted cylinder constraint and in Figure 12 for the extended cylinder constraint. A parametric CAD program facilitates rendering these overlay results as the base link can be geometrically constrained to remain horizontal while the cylinder length and the incline angle can be assigned specific values. During the overlay process, either the base link length or the rocker length could be chosen as the independent variable. In this case, a parametric CAD solution was obtained by making the base link a reference dimension while adjusting the value for the parameter that defines the length of the rocker. Thus, when the trends for the pairs of rocker and base lengths that satisfy the position constraints are plotted, the rocker length will be the independent variable and the base length will be the dependent variable. The choice is arbitrary. A graph of the overlay results shown in Figures 11 and 12 is made in Figure 13. The convergence analysis of the base length changes given in Table 3 for the overlay data reveals that an 8.000 inch rocker and a 13.798 inch base will satisfy the convergence criteria of less than 1% difference in base length for each of two positions.

For comparison, this problem was solved using general purpose multibody and optimization programs, MotionView (2004) and HyperStudy (2004). The solution involved defining a parametric model of the device then specifying a design variable that drives the search for an optimal solution. The computational solution to this problem required less than one minute on a conventional laptop computer by R. Jha (personal communication January 10, 2005).

Plotting Nonlinear Relationships to Find Solutions

It is a coincidence that only three guesses for the rocker lengths yielded an acceptable solution to the previous design problem. If other values for the rocker length were chosen, then the curves shown in Figure 13 would look more like curves rendered in Figure 14. In fact, solutions to the linkage synthesis problem can be found by simply graphing the nonlinear relationship between the rocker length and the base length shown in Figure 13. For the problem at hand, the analytical relationship between the rocker length and the base length is found by first employing the law of sines, then enforce the rule that the sum of angles in a triangle is 180, finally employ the law of sines again. The derivation is shown in Equations 8 through 10 where the law of sines and the angle rule of triangles are used to develop the relationships. In these equations, the term "Cylinder_length" and "Incline_Angle" take on the values of the two positional constraints when solving for the appropriate rocker-base length pairs. They are constants in the equations that generate the locus of points satisfying one set of positional constraints. Graphing the relationship between the base length and the rocker length for a given cylinder length and incline angle amounts to substituting Eq. 8 into Eq. 9, and then substituting the value from Eq. 9 into Eq. 10.

$$\text{Rocker\_Angle} = \sin^{-1}\left(\left(\frac{\text{Rocker\_Length}}{\text{Cylinder\_length}}\right)\sin(\text{Incline\_Angle})\right) \quad (8)$$

$$\text{Base\_Angle} = 180 - \text{Incline\_Angle} - \text{Rocker\_Angle} \quad (9)$$

$$\text{Base\_Length} = \left(\frac{\text{Cylinder\_length}}{\sin(\text{Incline\_Angle})}\right) \times \sin(\text{Base\_Angle}) \quad (10)$$

A graph of the rocker-base length pairs resulting from the two positional constraints is shown in Figure 14. Note the elliptical shape of the graphs indicate two possible solutions for this design problem. The first intersection was found with the preliminary overlay data. The linkage design from this solution is shown in Figure 15 where a rocker length of 8.000 and a base length of 13.798 inches are used. A second intersection occurs when the rocker is approximately 13.75 inches long. This second possible solution is rendered schematically in Figure 16. The design requirement that the maximum incline angle should be exactly 60^o is chosen, thus the minimum incline angle value will not be exactly 30^o. As previously discussed with the oven door problem, the solutions obtained during synthesis design are approximate so only one positional constraint can be met exactly.

Conclusion

Throughout this paper a graphing-based technique has been shown to facilitate the search for an optimal kinematic synthesis solution for problems that cannot be solved with straight forward geometric construction such as function generation type problems. In addition, the nonlinear mathematical relationships between links of a mechanism can be plotted to determine solutions to the synthesis problem. This also facilitates the rendering of an optimal design. This practice of graphing the complex trends observed while performing the overlay technique is another example of plotting complicated mathematical relationships to facilitate design as seen in other fields such as heat transfer calculations (Heisler charts), water pipe sizing (Moody charts), and pump system curves.

It is hoped that the practical examples shown in this article will motivate instructors of mechanism design courses to consider implementing exercises involving the kinematic synthesis of linkages. The synthesis activity complements the lessons of kinematic analysis and highlights the fact that the design of new mechanisms requires a different set of problem solving tools than that used for analysis alone.

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	Types of mathematics required for task	
	Analysis	**Synthesis**
Cams	Trigonometry	Trigonometry
Gears	Algegra	Algegra
Linkages (4-bar)	Trigonometry	Trigonometry and Nonlinear equation solution

Table 1. Mathematics used in mechanism design of cams, gears, and linkages

	Coupler Length (in)			
Crank Length (in)	Angle = 10, Opening = 0	Angle = 30, Opening = 4	Coupler difference (in)	Coupler difference % of average
	Closed_10_0	Opened_30_4		
4.0	15.135	12.825	2.310	16.52%
6.0	14.934	13.132	1.802	12.84%
10.0	15.329	14.573	0.756	5.06%
13.0	16.277	16.227	0.050	0.31%
13.3	**16.399**	**16.413**	**0.014**	**0.09%**
13.5	16.482	16.539	0.057	0.35%
16.0	17.689	18.228	0.539	3.00%
20.0	20.120	21.264	1.144	5.53%

Table 2. Crank and coupler pairs satisfying each design constraint (optimal solution in bold font)

	Base Length (in)			
Rocker Length (in)	Cylinder = 8" Incline = 30°	Cylinder = 12" Incline = 60°	Base difference (in)	Base difference % of average
4.0	11.210	13.489	2.279	18.45%
6.0	12.612	13.817	1.204	9.11%
8.0	**13.856**	**13.798**	**0.058**	**0.42%**
10.0	14.905	13.307	1.599	11.33%
12.0	15.684	12.000	3.684	26.61%

Table 3. Rocker and base pairs satisfying each design constraint (optimal solution in bold font)

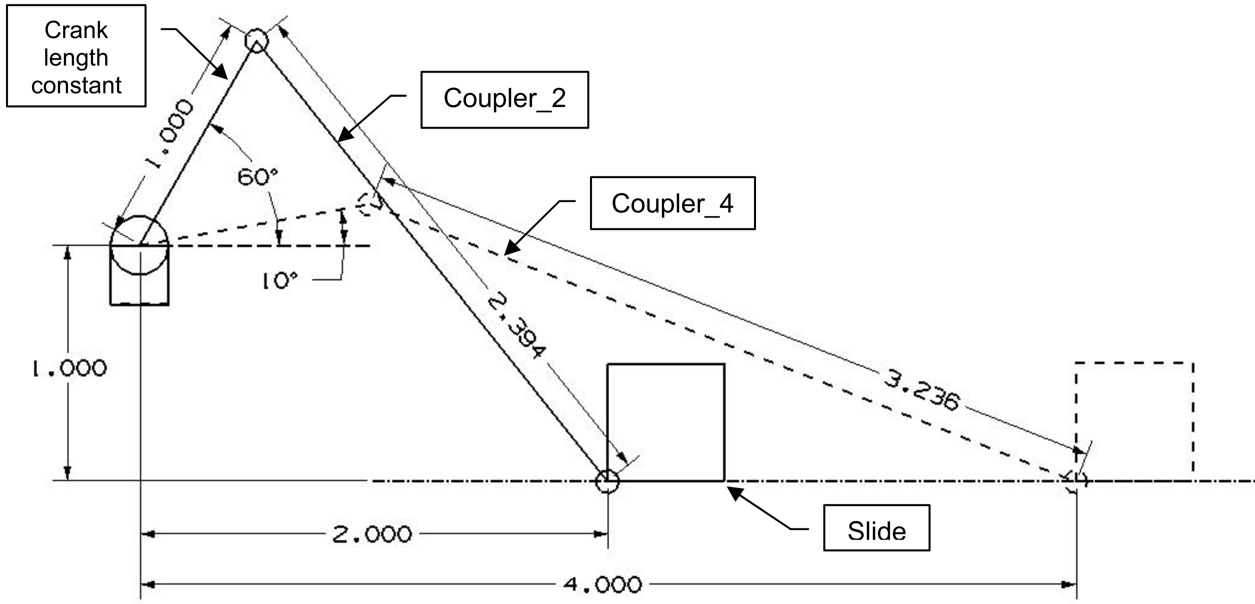

Figure 1. Schematic of crank slider mechanism

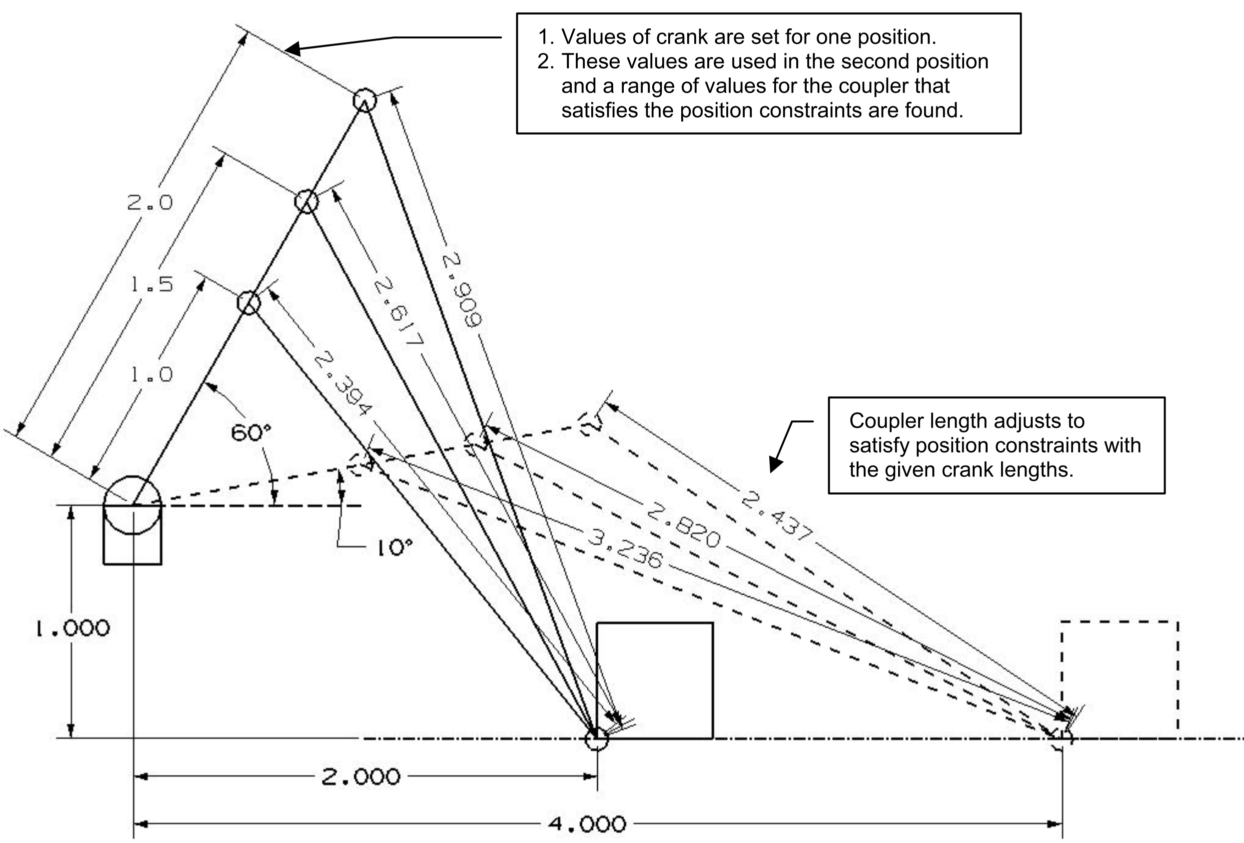

Figure 2. Overlay technique used to determine crank-coupler pair that solves design problem

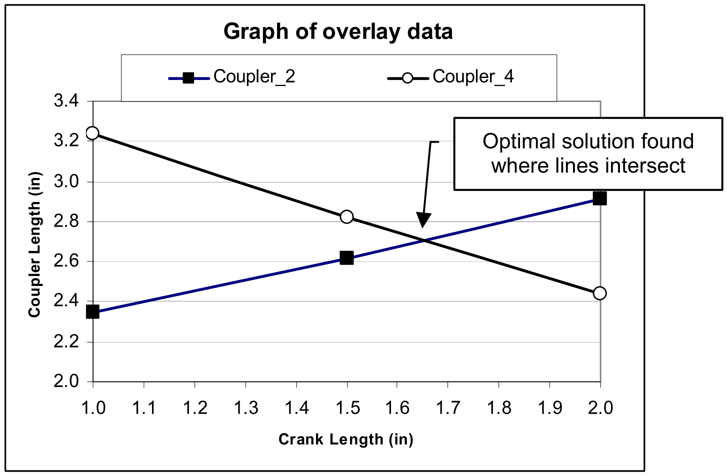

Figure 3. Optimal solution can be found by graphing results of overlay technique

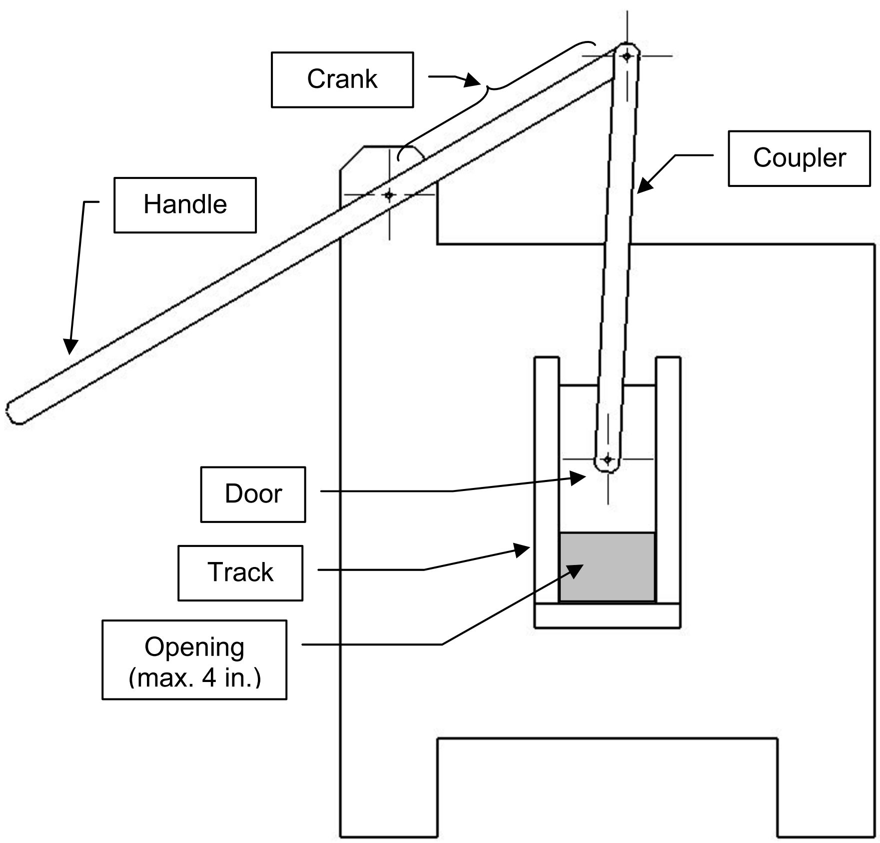

Figure 4. Schematic of oven with door opening mechanism

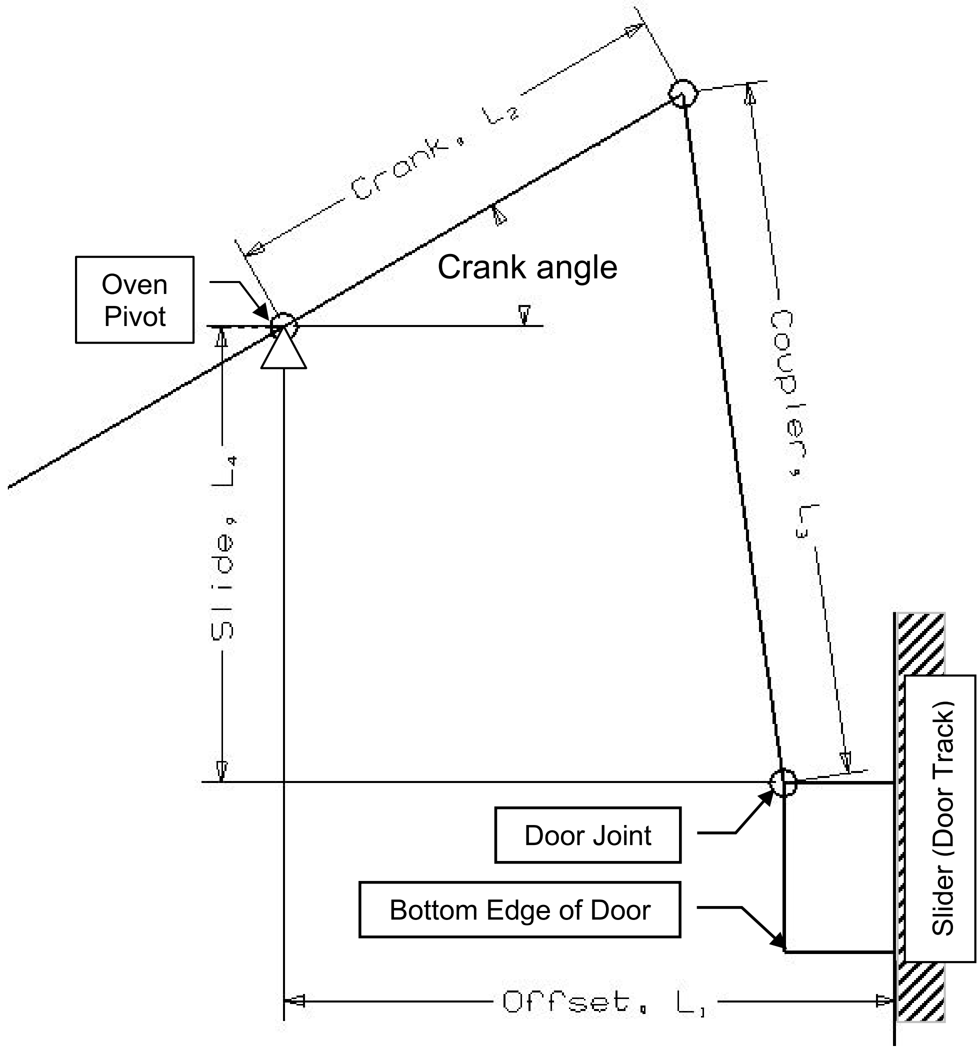

Figure 5. Kinematic diagram of oven door mechanism

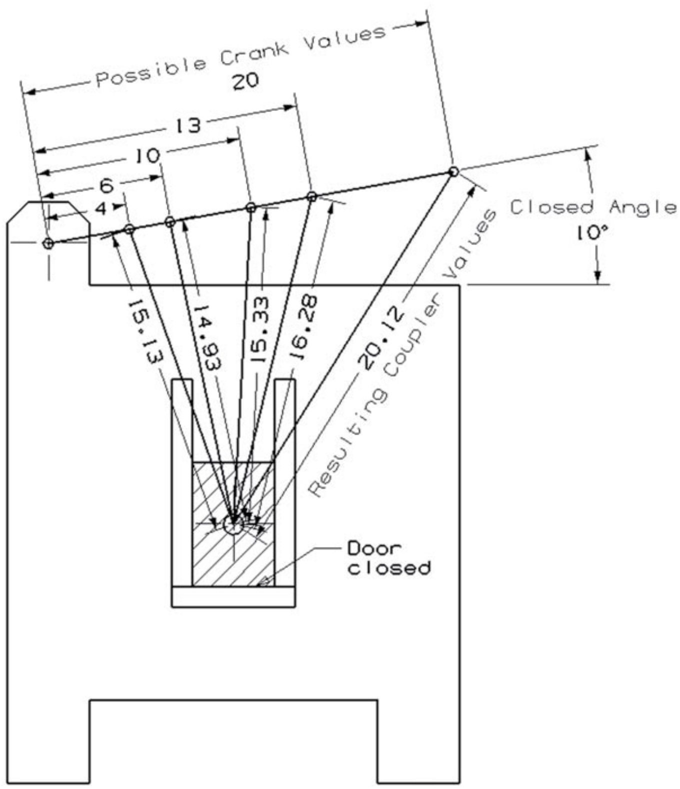

Figure 6. Representative pairs of crank and couplers satisfying the closed constraint

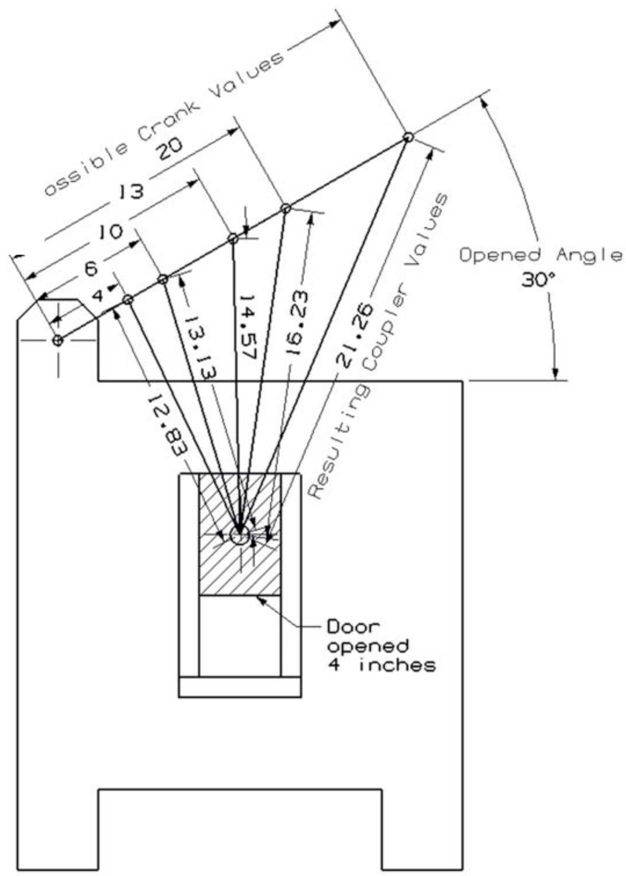

Figure 7. Representative pairs of crank and couplers satisfying the opened constraint

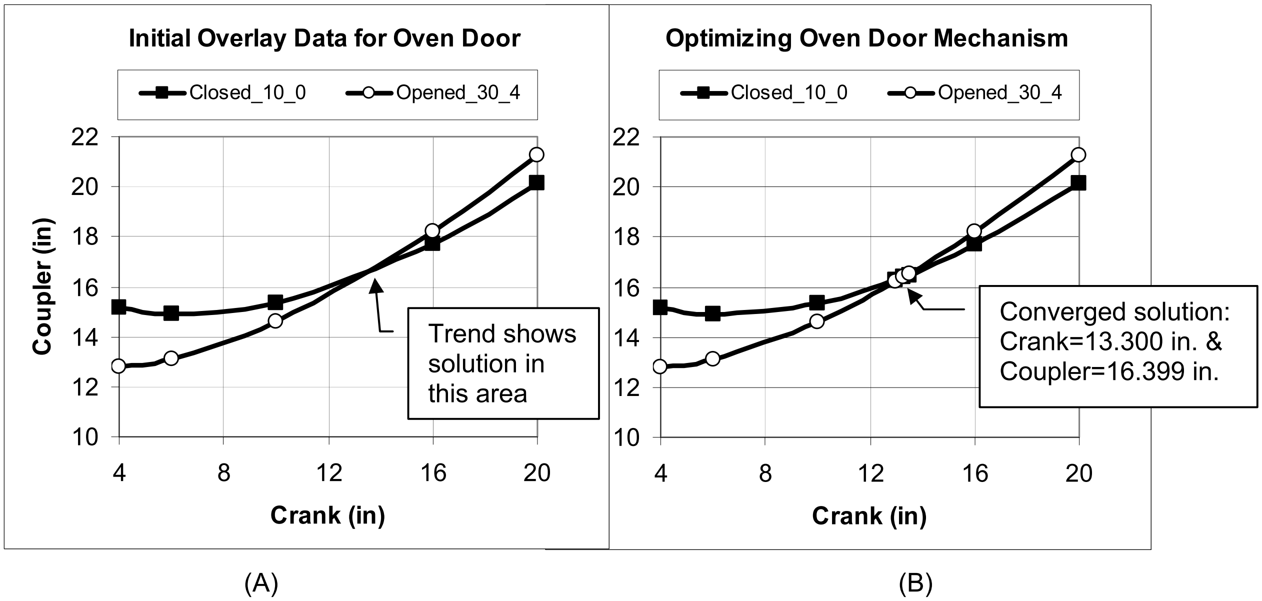

Figure 8. Graph of data in Table 2 illustrating the overlay trend

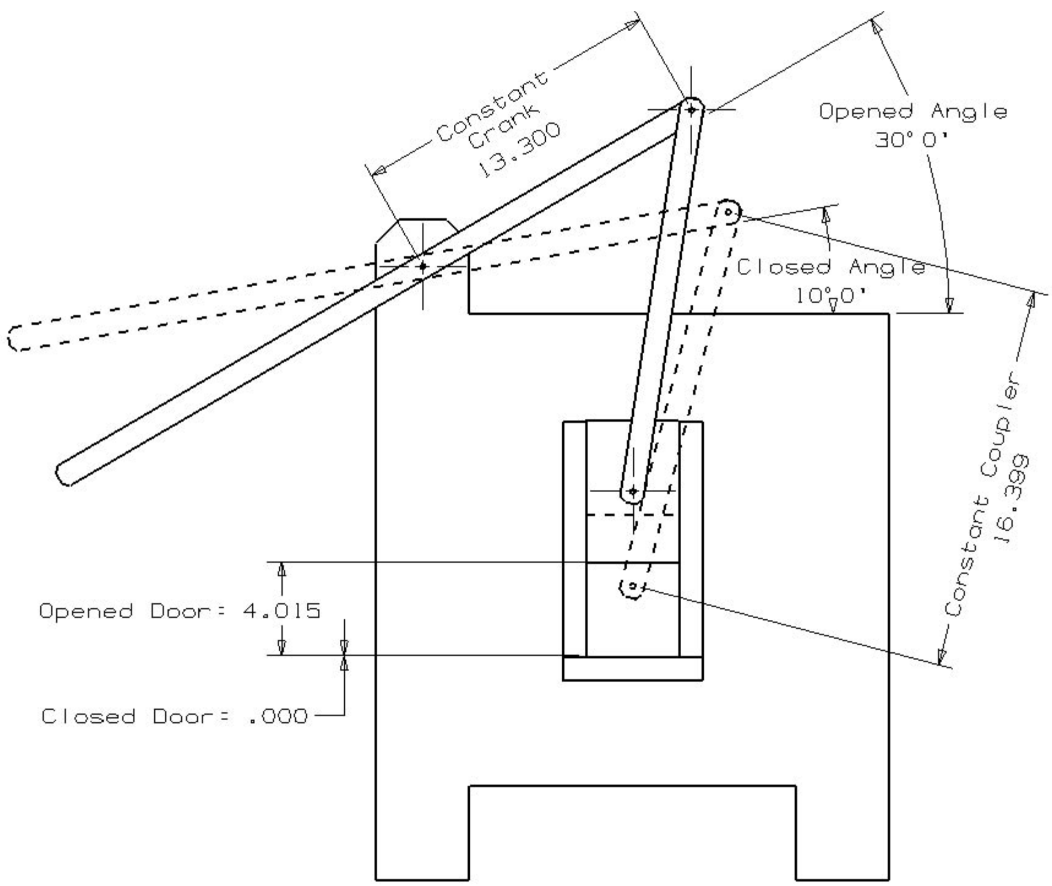

Figure 9. Overlay of the optimized door opening mechanism at the closed and fully opened positions

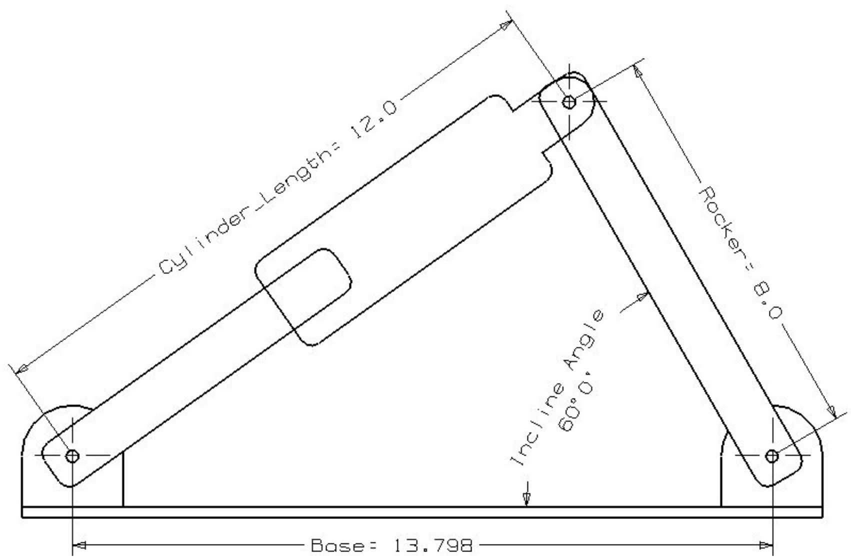

Figure 10. Schematic of Cylinder Incline Mechanism

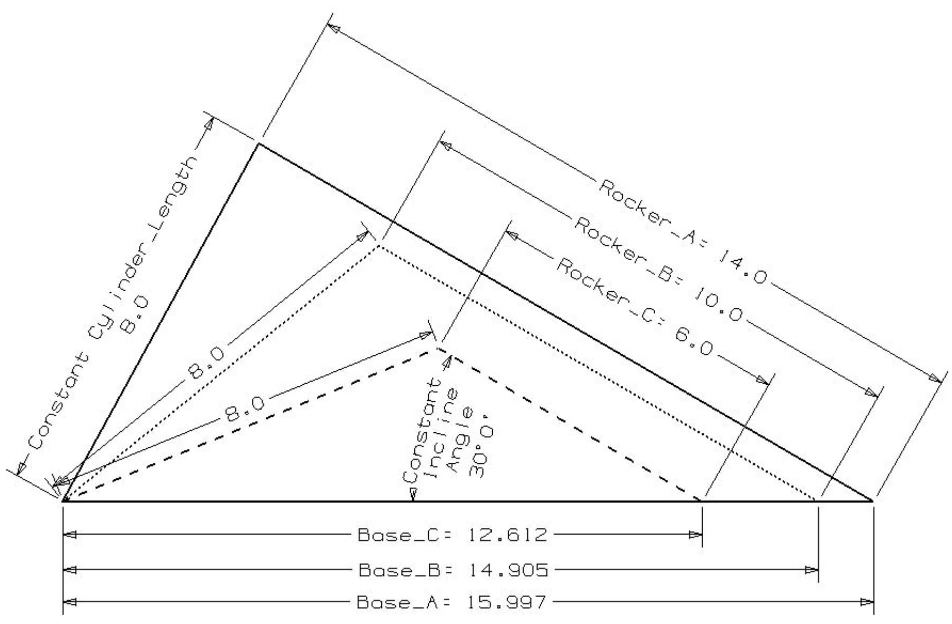

Figure 11. Overlay technique used to satisfy the retracted cylinder constraint

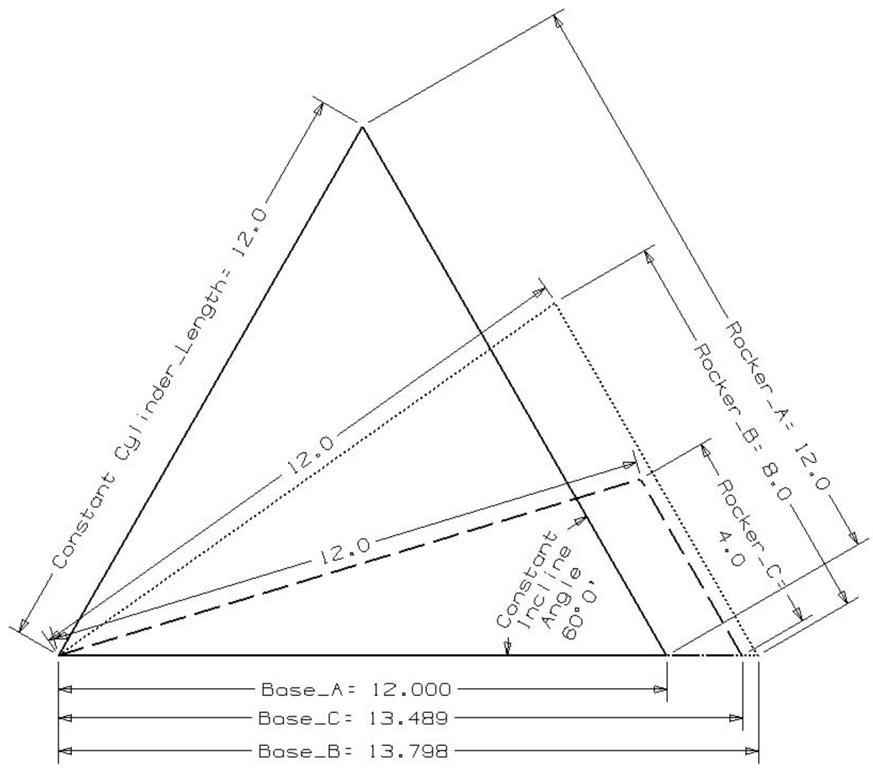

Figure 12. Overlay technique used to satisfy the extended cylinder constraint

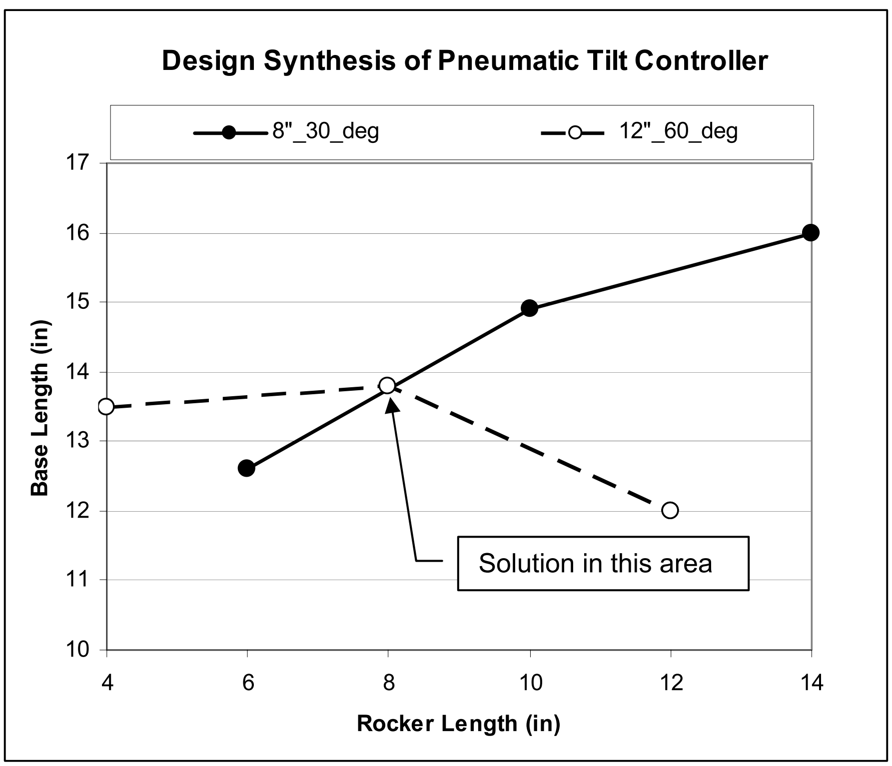

Figure 13. Graph of preliminary overlay results

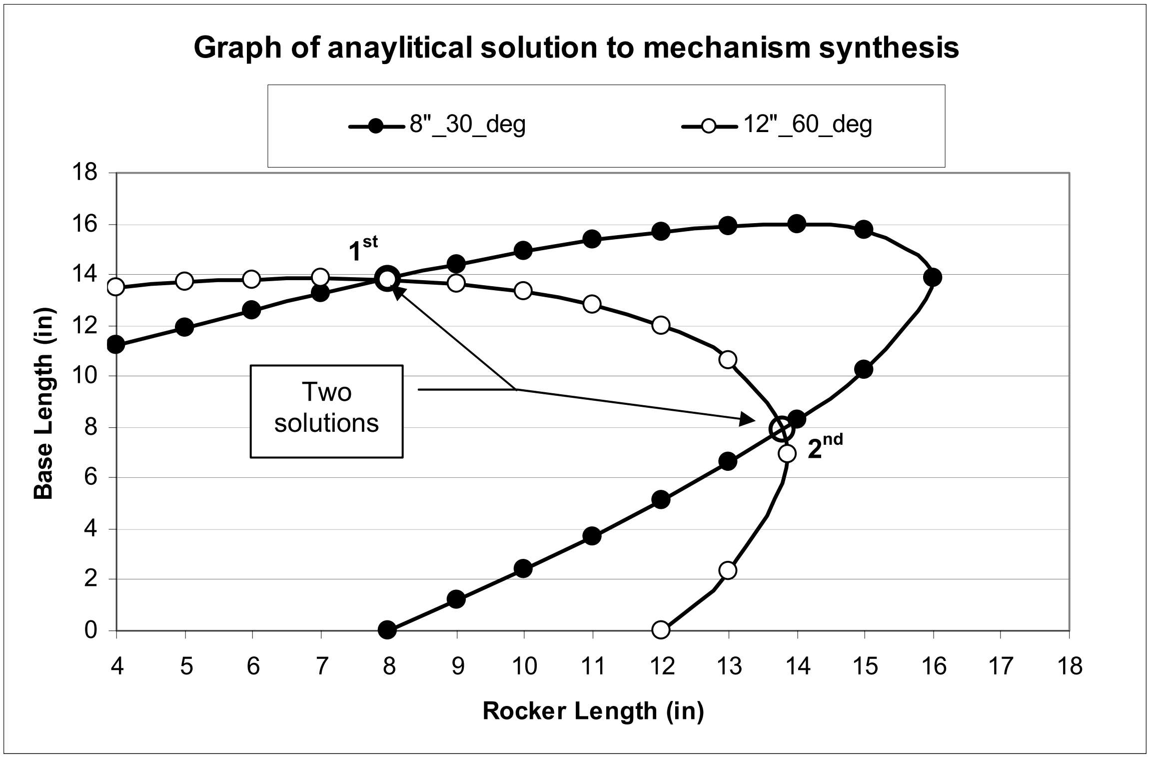

Figure 14. Graph illustrates two solutions possible for the tilt mechanism design

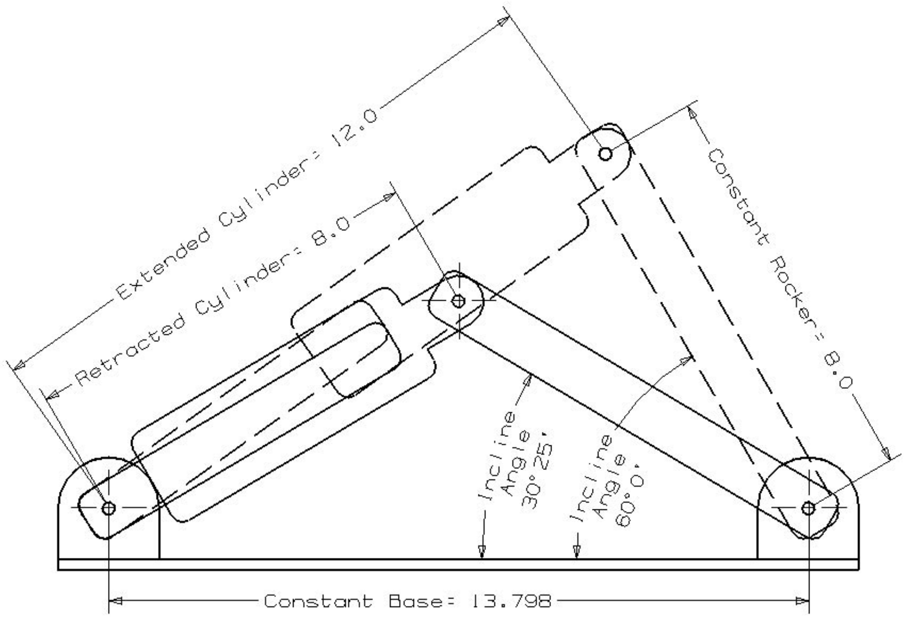

Figure 15. Overlay of 1st solution shown in Figure 14

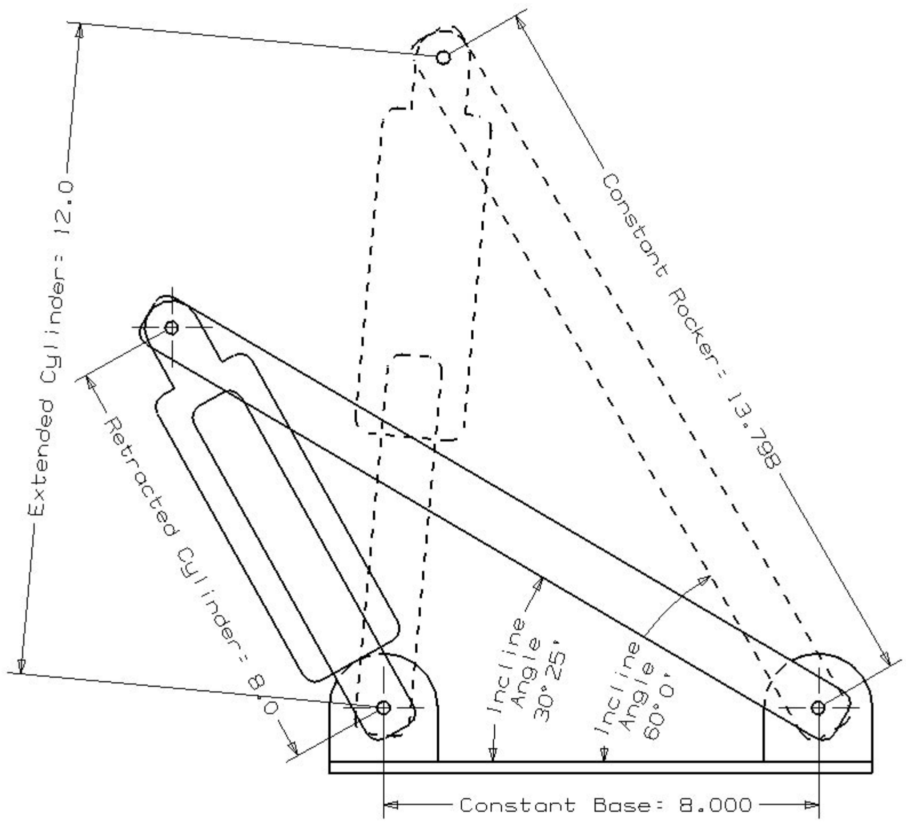

Figure 16. Schematic of 2nd solution shown in Figure 14