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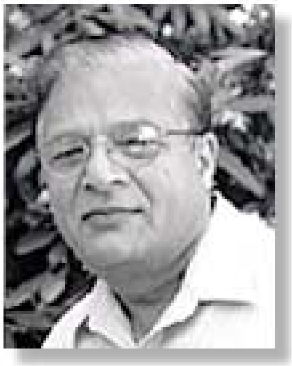

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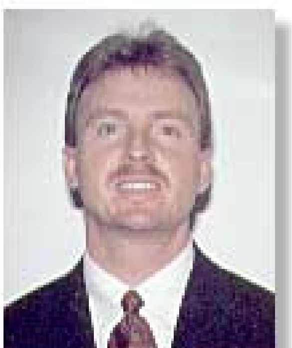

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Abstract

In planning a 2^{k-p} fractional factorial experiment, prior knowledge may enable an experimenter to pinpoint interactions which should be estimated free of the main effects and any other desired interactions. Taguchi (1987) gave a graph-aided method known as linear graphs associated with orthogonal arrays to facilitate the planning of such experiments. Since then the graph-aided method has been enhanced by various authors such as Li et al (1991), Wu and Chen (1992), Robinson (1993), and Wu and Hamada (2000). In this paper, we propose an algorithm for developing all possible non-isomorphic interaction graphs for combined arrays.

Introduction

Industrial Technologists (IT's) are frequently required to use designed experiments as problem solving and process improvement tools in their roles as "…technical and/or management oriented professionals…in business, industry, education, and government." (The National Association of Industrial Technology, 1997, [Online]). In fact, it is common practice for IT's to work with other communities of technical and managerial professionals as experimenters in the application of designed experiments in discrete part manufacturing, assembly, and process industries. Of primary importance to these experimenters is working with design, production, and quality personnel to identify and understand the many variables or factors associated with any type of industrial operation. It is particularly interesting to these experimenters to pinpoint those variables that most significantly influence or impact their industrial operations as well as any interactions among those variables. To pinpoint these most significant variables and their interactions, the IT's, engineers, and management team members who serve in the role of experimenters rely on the Design of Experiments (DOE) as the primary tool of their trade.

Within the branch of DOE known as classical methods, it is possible for experimenters to make use of their technical skills and in-depth knowledge of their particular industrial operations to design more effective, less expensive experiments using non-isomorphic interaction graphs. Accordingly, in this paper the authors will provide for readers a context for the application of non-isomorphic interaction graphs, formally define and describe these graphs as valuable tools, provide an algorithm for developing all possible graphs for combined arrays, provide for readers a detailed example developing and using these graphs, explain how to read and interpret the graphs, and pose the argument that IT's and other interested professionals can use non-isomorphic interaction graphs to do their jobs more effectively and efficiently.

Background

In planning a 2^{k-p} fractional factorial experiment, prior knowledge may enable experimenters to pinpoint interactions which should be estimated free of main effects and any other desired interactions. While working on applications of classical DOE methods, Taguchi (1987) introduced a graph-aided technique known as linear graphs associated with orthogonal arrays to facilitate the planning of such experiments, and as such, Taguchi's graph aided technique is not to be confused with what is commonly known as

Taguchi Methods. Linear graphs are graphical representations of the allocation of main effects and desired two-factor interactions to the columns of orthogonal arrays. Linear graphs facilitate the selection of an aliasing pattern that in turn enables experimenters to estimate all main effects and desired two-factor interactions. And it should be noted it is important for experimenters to be able to quantify the existence and magnitude of these main effects and interactions to support fact-based decision making regarding the experiment design and application.

Since 1987, researchers such as Li, Washio, Iida and Tanimoto (1991) have extended Taguchi's work on classical DOE applications of linear graphs by developing non-isomorphic linear graphs for an experiment involving eight (8) factors. In this scenario, a 2^k experiment design was crafted wherein no main effect was confounded with any other main effect or with any two-factor interactions, while two-factor interactions were confounded with each other. This type of experiment is commonly referred to as a resolution IV design and, in this case, the researchers used orthogonal arrays based on 16 experimental runs as the basis for the experiment and application of linear graphs. Wu and Chen (1992) then developed interaction graphs using the criterion of minimum aberration. Robinson (1993) followed by suggesting other modifications of Taguchi's linear graphs. The remainder of work related to interaction graphs available in the literature to date pertains to orthogonal arrays corresponding to a single set of factors. Taguchi, however, divided the factors into two categories, that is control factors and noise factors.

Relating Taguchi's Work To Non-Isomorphic Interaction Graphs

Taguchi introduced the technique of robust parameter design to reduce performance variation in products and processes by selecting the setting of control factors or design parameters so that performance is insensitive to noise factors such as environmental conditions, properties of raw materials and any other factors that are hard-to-control. Taguchi's technique, also known as Taguchi methods, uses an experimental design consisting of a cross product of two arrays, an inner array containing the control factors and an outer array containing the noise factors. The cross product of the inner and outer array often leads to a large number of observations that are generally very expensive to complete.

In an attempt to reduce the number of observations needed to support an experiment, and thus reduce costs, Welch et al (1990), Shoemaker et al (1991), Montgomery (1991), and Borkowski and Lucas (1997) suggested independently an alternative design to study both the control and noise factors by using a single array, called a combined array. A combined array structure enables experimenters to estimate both control-to-control and control-to-noise interactions with fewer observations. Miller et al (1993) used a combined array in an automobile experiment and demonstrated similar results could be obtained using the combined array approach and a much smaller number of runs as by using the cross product array. Since the noise factors are not usually controllable, main effects and two factor interactions of noise factors are less important than control-to-noise interactions, Chen et al (1993). More recently, Borror et al (2002) studied statistical designs for experiments involving noise factors

In practice, both control-to-control and control-to-noise interactions are normally important. Control-to-control interactions play an important role in product design and production processes while control-to-noise interactions play their role in product performance and are associated with variation. The structure of these interactions determines the nature of non-homogeneity of process variance that characterizes product design problems. Accordingly, a "good" product design is one where all desired control-to-control and control-to-noise interactions can be estimated by using a minimum number of runs. This can be achieved by using interaction graphs for two-level combined arrays.

In this paper, we give an algorithm for developing all possible non-isomorphic interaction graphs for a combined array of various two level fractional factorial designs. We do this in the context of classical DOE applications even though the topic could be applied to Taguchi Methods. These interaction graphs enable one to allocate the factors to the columns of orthogonal arrays, along with the estimation of main effects and desired control-to-control, control-to-noise interactions assuming all other interactions to be negligible.

"Non-Isomorphic" Defined

Having introduced the concept of linear interaction graphs in the preceding paragraphs, it is time to explain the nature and meaning of the concepts of isomorphic and non-isomorphic as they relate to interaction graphs. Figure 1 below is provided to help readers visualize the explanation.

As can be seen in Figure 1 (page 4), graph "A" depicts interactions among factorsV_{1-5}, graph "B" depicts interactions among factors U_{1-5}, and graph "C" depicts interactions among factors $W_{1-5.}$ Each graph displays a certain set of characteristics that describe relationships, or interactions, among the factors.

It is common practice during the design of an experiment for experimenters to assign factors so as to exhibit interactions if they are known to exist and are of interest. It is also common during the design of an experiment for experimenters to rename selected factors so as to exhibit different interactions within the same fractional factorial experiment in order to get more information from fewer experimental runs.

When experimenters rename the factors to observe different interactions within a fractional factorial experiment, and the renaming does not change any of the graph characteristics, the interaction graphs are considered to be isomorphic. Graph "A" and "B" of Figure 1 are thus isomorphic as the characteristics depicted before and after renaming of the factors are conceptually the same.

When experimenters rename the factors to observe different interactions within a fractional factorial experiment, and the renaming does change one or more of the graph characteristics, the interactions are considered to be non-isomorphic. Comparing either of graph "A" or "B" (i.e., the isomorphic interaction graphs) to graph "C" reveals the change of several characteristics and thus graphs "A" and "B" are non-isomorphic to graph "C."

An Algorithm For Developing A Non-Isomorphic Alias Structure For A Different Number Of Control And Noise Factors

In order to define a non-isomorphic alias structure for a given relation the following two criterion must be met within the alias structure where C = a control factor and N = a noise factor:

a) Count the number of clear CxC, CxN & NxN interactions.
b) Count the number of alias CxC with CxC; CxC with CxN & CxN with CxN interactions (aliasing of any two factor interaction NxN interaction is assumed to be clear two factor interaction).

For a given defined relation, we first construct an alias structure with a pre-defined number of control and noise factors. Next we rename the control and noise factors and observe the change in the alias structure on the basis of the criterion discussed above. It has been observed that the alias pattern changes with the renaming of factors and for different numbers of control and noise factors. This method gives us all possible non-isomorphic alias structure for different numbers of control and noise factors for a given defining relation.

Example

Consider a 2^{6-2} fractional factorial experiment with defining relation I=ABCE=BCDF. Suppose there are four (4) control factors and two (2) noise factors. This gives two non-isomorphic defining relations of same word length pattern, but of different alias structure. These defining relations are as follows:

a) I=ABCE=BCdf

AE	AB	AC	Bd	Bf	Ad	Af
BC	CE	BE	Cf	Cd	Ef	dE

b) I=ABCe=BCDf

Ae	AB	AC	BD	Bf	AD	Af
BC	Ce	Be	Cf	CD	-	De
Df						

An Algorithm For Developing Interaction Graphs For A Combined Array

The following steps are required for developing non-isomorphic interaction graphs. The method for developing the algorithm is based on the technique given by Li, Washio, Iida and Tanimoto (1991) and Wu and Chen (1992).

STEP 1: Consider a defining relation with specified number of control and noise factors.

STEP 2: Based on the defining relation allocate both control and noise factors to the columns of orthogonal arrays and developed alias structure.

STEP 3: Form a set of two-factor interactions by selecting one two-factor interaction from each aliased two-factor interactions along with all clear two-factor interactions.

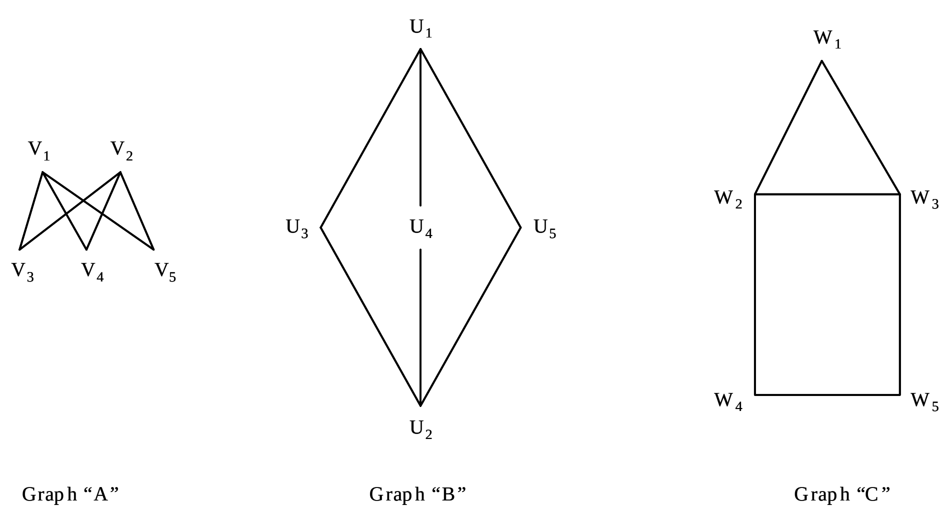
STEP 4: Calculate interaction matrix for finding total number of non-isomorphic interaction graphs, see Li, Washio, Iida and Tanimoto (1991). The two columns and rows of the interaction matrix are headed alphabetically (representing the control and the noise factors). Then the ij^{th} entry in the interaction matrix is 1 if the interaction between i^{th} row and j^{th} column belongs to the set formed in Step 3 and 0 otherwise.

STEP 5: Calculate the column total of the interaction matrix which are called patterns. These patterns represent the number of interactions associated with each factor, i.e., the number of lines (edges) starting from each factor (node).

STEP 6: Calculate extended patterns which are defined as $D_i = \Sigma d_{ij}$, where d_{ij}'s the pattern of all factors adjacent to i^{th} factor.

STEP 7: Arrange in descending order the patterns and extended patterns. Since there are two types of factors, divide the pattern in two groups, one corresponding to the control

Figure 1. Isomorphic vs. Non-Isomorphic Interaction Graphs



factors and the other corresponding to the noise factors.

STEP 8: Repeat the steps 4 to 7 for all combinations. Different combinations will give rise to non-isomorphic graphs if the patterns or the extended patterns are distinct.

STEP 9: Corresponding to each distinct combination, develop an interaction graph.

The clear interaction between control-to-control factors and control-to-noise factors are represented by solid lines and the eligible, but not clear interaction between factors are represented by broken lines.

We explain in greater detail the algorithm steps for developing non-isomorphic graphs with the help of an example as was discussed in Keats and Montgomery (1991).

Example

In casting aircraft and jet turbine engine parts, a preliminary goal is to determine an appropriate alloy chemistry for the material. The intent is to achieve certain physical properties, such as high ultimate tensile strength, which is achieved by varying the alloy elements of materials. In this experiment, Keats and Montgomery (1991) considered seven factors (each at 2 levels) of which four are control factors namely (1) aluminum (2) tritanium (3) chromium and (4) silicon and the rest are noise factors, i.e. (5) heat treatment (6) temperature and (7) oxygen level. Now we consider a 2^{7-2} design with 32 runs, having WLP (word length pattern) as (000455), see Chen et al (1993). This will give a resolution IV design. It can be seen there are four non-isomorphic defining relations of WLP (000455) which are as follows:

(1) I=BCdeF=ACdeg=ABFg
(2) I=BCdeF=AcdeG=ABFG
(3) I=BCDef=ACDeg=ABfg
(4) I=bCDEf=ACDEg=Abfg

Now consider the first defining relation i.e., I=BCdeF=ACdeg=ABFg. The alias structure for the given defining relation is as shown in Table 1.

There are eight (8) possible combinations of eligible, but not clear two-factor interactions. Consider one of the combinations annexing the clear two-factor interactions as:

AC Ad Ae BC Bd Be Cd
Ce CF Cg dF eF Fg Bg
BF

The interaction matrix, pattern and extended pattern for the above combination is shown in Table 2.

The sorted pattern and extended pattern are [3 5 5 6 3 4 4] and [14 22 22 24 16 19 19] respectively.

Proceeding in this manner, we get three distinct patterns and extended patterns that are listed in Table 3 (page 6). These patterns and extended patterns give rise to three non-isomorphic interaction graphs that are shown in Figure 2 (page 7) as solid lines indicating clear interactions between factors and as dotted lines indicating possible, but not clear, interactions between other factors.

Similarly, we get two non-isomorphic interaction graphs corresponding to the defining relation I=BcdeF=AcdeG, two non-isomorphic interaction graphs corresponding to the defining relation I=BCDef=ACDeg and one non-isomorphic interaction graph for I=bCDEf=ACDEg. The list of patterns and extended patterns are given in Table 4 (page 6).

A detailed list showing the number of non-isomorphic interaction graphs for various two level fractional factorial designs composed of a different number of control factors and noise factors is given in the Appendix. A complete catalogue of interaction graphs is available with the authors.

Use And Interpretation Of Non-Isomorphic Interaction Graphs

Designing an experiment using only few non-isomorphic interaction graphs is very simple and efficient. This technique allows us at a glance to decide how to allocate different control as well as noise factors to different columns of the treatment matrix such that the desired interaction can be estimated with as few experimental runs as possible. For instance, in the above

Table 1. Alias Structure

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	B	AB Fg	C	AC	BC		d	Ad	Bd		Cd		eF	

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
e	Ae	Be		Ce		dF			Cg	CF		BF Ag	g	F	AF Bg

Table 2. Interaction Matrix

	A	B	C	F	d	e	g
A	0	0	1	0	1	1	0
B	0	0	1	1	1	1	1
C	1	1	0	1	1	1	1
F	0	1	1	0	1	1	1
d	1	1	1	1	0	0	0
e	1	1	1	1	0	0	0
g	0	1	1	1	0	0	0
Pattern	3	5	6	5	4	4	3
Extended	14	22	24	22	19	19	16
Pattern							

example, if we are interested in estimating the interaction between aluminum and tritanium, then we can easily see from the graphs in Figure 2 that we must assign aluminum and tritanium to columns (A,C), (B, C), or (C, F).

By extension, as experimenters design fractional factorial experiments, as opposed to more costly and time consuming experiments such as full factorial designs, non-isomorphic interaction graphs become the primary means for identifying the interactions of interest, and we visualize those interactions of interest as the solid lines in the interaction graphs. And since we can readily identify the existence or non-existence of a desired interaction, as experimenters, IT's can readily focus their attention on renaming factors as needed to obtain the interactions of interest to study these interactions with as few experimental runs as possible – saving time and money, getting better information for use in the problem solving or process improvement process, and reducing the chance of experimental error or information overload.

Table 3. List Of Distinct Patterns And Extended Patterns
For 2^{7-2} Designs With I=BCdeF=ACdeg

Combinations	Patterns	Extended Patterns
AC Ad Ae BC Bd Be Cd Ce CF Cg dF eF Fg Bg BF	3 5 5 6 3 4 4	14 22 22 24 16 19 19
AC Ad Ae BC Bd Be Cd Ce CF Cg dF eF Fg AF BF	4 4 6 6 2 4 4	20 20 24 24 12 20 20
AC Ad Ae BC Bd Be Cd Ce CF Cg dF eF AB AF BF	5 5 5 6 1 4 4	24 24 24 24 6 21 21

Table 4. List Of Distinct Patterns And Extended Patterns

Combinations	Patterns	Extended Patterns
Ac Ad Ae Bc Bd Be cF cG dF dG eF eG FG BG Bf	3 5 5 5 4 4 4	12 22 22 22 18 18 18
Ac Ad Ae Bc Bd Be cF cG dF dG eF eG FG BG AG	4 4 4 6 4 4 4	18 18 18 24 18 18 18

List Of Distinct Patterns And Extended Patterns
For 2^{7-2} Designs With I=BCDef=ACDeg

Combinations	Patterns	Extended Patterns
AC AD Ae BC BD Be CD Ce Cf CgDe Df Dg AB Bg Bf	4 6 6 6 3 3 4	22 26 26 26 18 18 22
AC AD Ae BC BD Be CD Ce Cf Cg De Df Dg AB Bg Ag	5 5 6 6 2 4 4	25 25 26 26 12 22 22

List Of Distinct Patterns And Extended Patterns
For 2^{7-2} Designs With I=bCDEf=ACDEg

Combinations	Patterns	Extended Patterns
AC AD AE bC bD bE CD CE Cf Cg DE Df Dg Ef Eg Ab Af Ag	6 6 6 6 4 4 4	30 30 30 24 24 24

Conclusions

It is important to note that IT's are members of technical as well as managerial communities. As members of these communities, IT's have as their primary responsibilities, in many cases, completing the complex analyses needed to better understand industrial operations in the context of problem solving and process improvement or making informed decisions based on the results of experimental-based analysis. In order to do their jobs more efficiently and effectively IT's need at least a familiarization with high-level quantitative tools such as discussed in this paper to facilitate the analysis activity. Non-isomorphic interaction graphs represent an enhancement to the IT's tool box enabling IT's to design experiments that produce valuable data and information from as few experimental runs as possible – certainly with fewer runs than would be possible without using the graphs.

Since the financial implications of completing such quantitative analyses with fewer rather than more experimental runs can be substantial, IT's should, at the very least, be aware of the tools and their application to industrial operations. Further, knowing the mechanics of how to actually apply these tools can only help IT's contribute to the long-term competitiveness and survivability of their academic or industrial employers.

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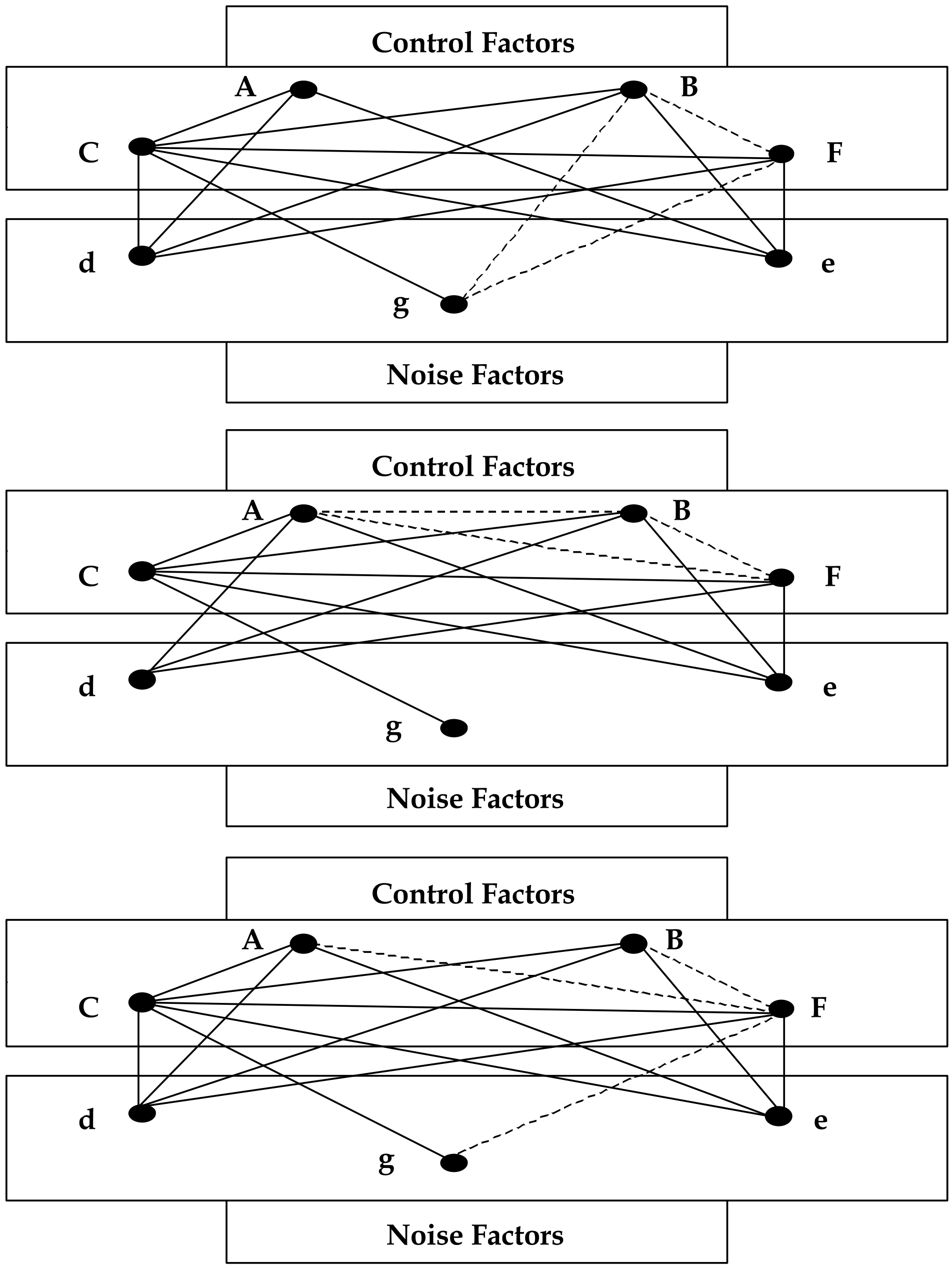
Figure 2. I = BCdeF = ACdeg



Appendix

Table 1. Number Of Non-Isomorphic Interaction Graphs For Various 2 Level Fractional Factorial Designs With Different Number Of Control Factors And Noise Factors.

Design	Defining Relation	No. of Control Factors	No. of Noise Factors	No. of Non-Isomorphic Interaction Graphs
2^{4-1}	I=ABCd	3	1	3
2^{4-1}	I=Abcd	2	2	2
2^{5-1}	I=ABCe	4	1	3
2^{5-1}	I=ABce	3	2	3
2^{6-2}	I=ABCE=BCDf	5	1	27
2^{6-2}	I=ABCE=BCdf	4	2	14
2^{6-2}	I=ABCe=BCDf	4	2	26
2^{6-2}	I=ABce=BcDf	3	3	4
2^{6-2}	I=ABCe=BCdf	3	3	12
2^{7-2}	I=BCDeF=ACDeG	6	1	2
2^{7-2}	I=BCDEF=ACDEg	6	1	3
2^{7-2}	I=BcdeF=AcdeG	5	2	2
2^{7-2}	I=BCDeF=ACDeg	5	2	3
2^{7-2}	I=BCDEf=ACDEg	5	2	2
2^{7-2}	I=BCdeF=ACdeg	4	3	3
2^{7-2}	I=BcdeF=AcdeG	4	3	2
2^{7-2}	I=BCDef=ACDeg	4	3	2
2^{7-2}	I=BcDEf=AcDEg	4	3	1
2^{7-3}	I=ABCE=BCDF=ACDg	6	1	1
2^{7-3}	I=ABCE=BCDf=ACDg	5	2	1
2^{7-3}	I=ABCe=BCDf=ACDg	4	3	84
2^{7-3}	I=ABCE=BCdf=ACdg	4	3	28
2^{8-3}	I=BCDEF=ACDEG=ABDEh	7	1	27
2^{8-3}	I=BCDeF=ACDeG=ABDeH	7	1	7
2^{8-3}	I=BCDEF=ACDEg=ABDEh	6	2	26
2^{8-3}	I=BcDEF=AcDEG=ABDEh	6	2	14
2^{8-3}	I=BCDeF=ACDeG=ABDeh	6	2	30
2^{8-3}	I=BCdeF=ACdeG=ABdeH	6	2	7
2^{8-3}	I=BCDEf=ACDEg=ABDEh	5	3	3
2^{8-3}	I=BcDEF=AcDEg=ABDEh	5	3	12
2^{8-3}	I=BCDeF=ACDeg=ABDeh	5	3	26
2^{8-3}	I=BcDeF=AcDeG=ABDeh	5	3	14
2^{8-3}	I=BCdeF=ACdeG=Abdeh	5	3	30
2^{8-3}	I=bcDEF=AcDEg=AbDEh	4	4	4
2^{8-3}	I=BcDEf=AcDEg=ABDEh	4	4	2
2^{8-3}	I=BCDef=ACDeg=ABDeh	4	4	4

Design	Defining Relation	No. of Control Factors	No. of Noise Factors	No. of Non-Isomorphic Interaction Graphs
2^{8-3}	I=BcDeF=AcDeg=ABDeh	4	4	12
2^{8-3}	I=BCdeF=ACdeg=ABdeh	4	4	26
2^{8-3}	I=BcdeF=AcdeG=ABdeh	4	4	14
2^{8-4}	I=ABCE=BCDF=ACDG=ABDh	7	1	125
2^{8-4}	I=ABCE=BCDF=ACDg=ABDh	6	2	241
2^{8-4}	I=ABCE=BCDf=ACDg=ABDh	5	3	253
2^{8-4}	I=ABCE=BCdf=ACdg=ABdh	4	4	38
2^{8-4}	I=ABCe=BCDf=ACDg=ABDh	4	4	125
2^{9-3}	I=ABCDg=ACEFH=CDEFJ	8	1	6
2^{9-3}	I=ABCDG=ACEFH=CDEFj	8	1	8
2^{9-3}	I=ABCDg=ACEfH=CDEfJ	7	2	6
2^{9-3}	I=ABCDg=ACEFH=CDEFj	7	2	8
2^{9-3}	I=ABCDG=ACEFh=CDEFj	7	2	3
2^{9-3}	I=ABCDg=ACefH=CDefJ	6	3	2
2^{9-3}	I=ABCDg=ACEfH=CDEfj	6	3	4
2^{9-3}	I=ABCDg=ACEFh=CDEFj	6	3	2
2^{9-3}	I=ABCdG=ACEFh=CdEFj	6	3	1
2^{9-3}	I=ABcDg=AcefH=cDefJ	5	4	2
2^{9-3}	I=ABCDg=ACefH=CDefj	5	4	4
2^{9-3}	I=ABCDg=ACEfh=CDEfj	5	4	4
2^{9-3}	I=ABCdg=ACEFh=CdEFj	5	4	1
2^{9-3}	I=aBCdG=aCEFh=CdEFj	5	4	1
2^{9-4}	I=BCDEF=ACDEG=ABDEH=ABCEj	8	1	395
2^{9-4}	I=BCDeF=ACDeG=ABDeH=ABCeJ	8	1	66
2^{9-4}	I=BCDEF=ACDEG=ABDEh=ABCEj	7	2	971
2^{9-4}	I=BcdEF=ACdEG=ABdEH=ABCEj	7	2	310
2^{9-4}	I=BCDeF=ACDeG=ABDeH=ABCej	7	2	442
2^{9-4}	I=BCDEF=ACDEg=ABDEh=ABCEj	6	3	184
2^{9-4}	I=BcdEF=ACdEG=ABdEh=ABCEj	6	3	55
2^{9-4}	I=BCDeF=ACDeG=ABDeh=ABCej	6	3	593
2^{9-4}	I=BcdeF=ACdeG=ABdeH=ABCej	6	3	190
2^{9-4}	I=BcdEf=ACdEg=AbdEh=ABCEJ	5	4	12
2^{9-4}	I=BcdEF=ACdEg=AbdEh=ABCEj	5	4	74
2^{9-4}	I=BCDeF=ACDeg=ABDeh=ABCej	5	4	186
2^{9-4}	I=BcdEF=AcdEG=ABdEh=ABcEj	5	4	70
2^{9-4}	I=BcdeF=ACdeG=ABdeh=ABCej	5	4	197
2^{10-4}	I=ABCDG=ACEFH=CDEFJ=ABCEk	9	1	32
2^{10-4}	I=ABCDG=ACEFH=CDEFj=ABCEK	9	1	107
2^{10-4}	I=ABcDG=AcEFH=cDEFJ=ABcEK	9	1	90
2^{10-4}	I=ABCDg=ACEFH=CDEFj=ABCEK	8	2	150

Design	Defining Relation	No. of Control Factors	No. of Noise Factors	No. of Non-Isomorphic Interaction Graphs
2^{10-4}	I=ABCDg=ACeFH=CdeFJ=ABCeK	8	2	39
2^{10-4}	I=ABcDG=AcEFH=cDEFj=ABcEK	8	2	110
2^{10-4}	I=ABCDG=ACEFh=CDEFJ=ABCEk	8	2	51
2^{10-4}	I=ABCDG=ACEFH=CDEFj=ABCEk	8	2	132
2^{10-4}	I=ABCDG=ACEFh=CDEFj=ABCEK	8	2	102
2^{10-4}	I=ABCDG=ACEFh=CDEFj=ABCEk	7	3	55
2^{10-4}	I=ABCDg=ACEFH=CDEFj=ABCEk	7	3	148
2^{10-4}	I=ABCDg=ACEfH=CDEfj=ABCEK	7	3	88
2^{10-4}	I=ABCDG=ACEfh=CDEfJ=ABCEk	7	3	17
2^{10-4}	I=ABCDG=ACEfh=CDEfj=ABCEK	7	3	34
2^{10-4}	I=ABCDg=ACeFH=CDeFj=ABCeK	7	3	99
2^{10-4}	I=ABcDg=AcEFH=cDEFj=ABcEK	7	3	150
2^{10-4}	I=ABCDg=ACeFH=CDeFJ=ABCek	7	3	20
2^{10-4}	I=ABcDg=AceFH=cDeFJ=ABceK	7	3	39
2^{10-4}	I=ABCdG=ACEFh=CdEFJ=ABCEk	7	3	7
2^{10-4}	I=ABCDg=ACEFh=CDEFj=ABCEk	6	4	19
2^{10-4}	I=ABCDg=ACEfH=CDEFj=ABCEK	6	4	52
2^{10-4}	I=ABCdG=ACEFh=CdEFj=ABCEk	6	4	5
2^{10-4}	I=ABCDg=ACeFH=CDeFj=ABCek	6	4	19
2^{10-4}	I=ABCDG=ACEFh=CDEfj=ABCEk	6	4	15
2^{10-4}	I=ABcDg=AcEfH=cDEfj=ABCEK	6	4	96
2^{10-4}	I=ABCDg=ACeFH=CDeFj=ABCek	6	4	19
2^{10-4}	I=ABcDg=AcEfh=cDEfJ=ABcEK	6	4	18
2^{10-4}	I=ABcDg=AceFH=cDeFj=ABceK	6	4	99
2^{10-4}	I=ABCDg=ACefH=CDefJ=ABCek	6	4	9
2^{10-4}	I=ABcDg=AceFH=cDeFJ=ACcek	6	4	20
2^{10-4}	I=ABCdg=ACeFH=CdeFJ=ABCEk	6	4	19
2^{10-4}	I=ABCDg=ACefh=CDefj=ABCeK	5	5	89
2^{10-4}	I=ABcDg=AcEfH=cDEfj=ABcEk	5	5	165
2^{10-4}	I=ABCDg=ACEfh=CDEfj=ABCEk	5	5	225
2^{10-4}	I=ABcDg=AcefH=cDefj=ABceK	5	5	211
2^{10-4}	I=ABCDg=ACEfH=CDefj=ABCek	5	5	285
2^{10-4}	I=ABCdg=ACEFh=CdEFj=ABCEk	5	5	209
2^{10-4}	I=ABcDg=AceFH=cDeFj=ABcek	5	5	128
2^{10-4}	I=ABCDg=ACefh=CDefJ=ABCek	5	5	4
2^{10-4}	I=ABCdg=ACeFh=CdeFJ=ABCek	5	5	5
2^{10-4}	I=ABcDg=AcefH=cDefJ=ABcek	5	5	9
2^{10-4}	I=ABCdg=ACeFH=CdeFj=ABCek	5	5	15
2^{10-4}	I=ABcdg=AceFH=cdeFJ=ABcek	5	5	19
2^{10-4}	I=AbCDg=ACefH=CDefJ=AbCek	5	5	2