46th Annual Review of Progress
in Quantitative Nondestructive Evaluation
QNDE2019
July 14-19, 2019, Portland, OR, USA

QNDE2019-6781

SEMI-ANALYTICAL FINITE ELEMNET BASED GUIDED WAVE MODELLING AND APPLICATIONS IN NDE, SEISMOLOGY AND OIL & GAS

Peng Zuo, Zheng Fan[1]
School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 637798, Singapore

Abstract

Guided waves have been widely used in different fields of engineering, such as non-destructive evaluation, seismology and oil & gas. Successful applications of guided waves are dependent upon thorough understanding of their modal properties. However, the prediction of the behavior of guided waves could be challenging for complicated waveguides, for example embedded/immersed waveguides with significant energy leakage, anisotropy waveguides with mode coupling effect and prestressed waveguide with acoustoelastic behavior. In this paper, we develop a generalized tool to study the modal properties of complex waveguides using semi-analytical finite element (SAFE) method. It is based on acoustoelastic equations with an option to add perfectly matched layers for open waveguides. The model is implemented into a commercial software package, providing easy access for a wide range of researchers. The model is first validated by well-established solutions in the literature, and then applied to a few practical cases in different fields, demonstrating its great potential in guided wave applications.

Keywords: SAFE model, open waveguides, surface wave, anisotropy, prestress

1. INTRODUCTION

Guided waves have been widely used in various branches of engineering scenarios, such as non-destructive evaluation (NDE), where guided waves are used to inspect defects; seismology, in which elastic surface waves are applied for mantle tomography and oil & gas, where both Stoneley wave and pseudo Rayleigh wave are employed for borehole acoustic logging. Successful applications of guided waves mostly rely on the thorough understanding of modal properties of the guided waves. However, precise prediction of guided wave behavior is difficult for complex waveguides. The complications include waveguides being embedded/immersed in another media leading to significant energy leakage, material elastic anisotropic nature causing mode coupling effect and prestress in the media resulting in acoustoelasticity.

A number of techniques have been proposed to provide a potential to address the complications. For example, artificial surrounding layers[1] (e.g. absorbing layer, boundary element and perfectly matched layer) were developed to simulate the infinite surrounding media within a finite domain. A six-dimensional formalism was proposed by Stroh[2] to study Rayleigh wave propagation in anisotropic media, and perturbation theory[3] was commonly used to study the effect of prestress on guided wave propagation. However, most of the studies mentioned above are only applicable to specific cases, and a comprehensive method still remains to be unachieved.

In this paper, a semi-analytical finite element (SAFE) based guided wave model is developed in order to study the modal properties of guided waves under complicated conditions. The model starts with acoustoelastic equations and combines perfectly matched layer, which make it possible to simulate open waveguides. The model is implemented into a commercial software package providing an easily accessible approach for the study of guided waves.

2. MATHEMATICAL FRAMEWORK

The mathematical model starts with acoustoelastic equations and combines the governing equations of perfectly matched layer.

2.1 Acoustoelastic equations

By using the acoustoelastic equations, three assumptions are adopted: (1) the material is characterized by Murnaghan material mode, where both second and third order elastic constants are considered; (2) the pre-deformation is small and the material remains elastic so that the relationship between the initial stress and initial strain can be approximated by Hooke's law; (3) the amplitude of the elastic wave propagation in the material is much smaller than the pre-deformation.

The governing equation of the acoustoelastic wave with respect to the initial state is given by

$$t_{KL}^{i}\frac{\partial^2 u_I}{\partial X_K \partial X_L}+\frac{\partial}{\partial X_L}\left(C_{ILMK}\frac{\partial u_M}{\partial X_K}\right)=\rho^i\frac{\partial^2 u_I}{\partial t^2} \quad (1)$$

[1] Contact author: zfan@ntu.edu.sg

where ρ^i is the density in the initial frame which is a function of initial strains and it is related to ρ^0 and the dilatation e^i_{NN} by $\rho^i = \rho^0\left(1-e^i_{NN}\right)$; u is the incremental displacement from the initial state to the final state; t^i_{KL} demonstrates the component of the component of the Cauchy stress tensor for the initial state. The elastic constant tensor C_{ILMK} can be expressed by

$$C_{IJKL} = c_{IJKL} + c_{MJKL}\frac{\partial u^i_I}{\partial X_M} + c_{IMKL}\frac{\partial u^i_J}{\partial X_M} + c_{IJML}\frac{\partial u^i_K}{\partial X_M} + c_{IJKM}\frac{\partial u^i_L}{\partial X_M} - c_{IJKL}e^i_{NN} + c_{IJKLMN}e^i_{MN}$$

(2)

where c_{IJKL} and c_{IJKLMN} represent the second and third order elastic constants, respectively; u^i_I demonstrates the initial displacement from the natural state to the initial state and the e^i_{MN} is the component of the initial Cauchy strain tensor.

2.2 Perfectly matched layer

The concept of PML was first created by Bérenger[4] in the context of electromagnetic waves and then it was shown that the PML equations result from a complex-valued coordinate stretching in the electromagnetic wave equations. The same ideas are immediately applicable to elastic wave equations. The stretched coordinates in the waveguide are defined as

$$\begin{cases} \tilde{x}_1\left(x_1\right) = \int_0^{x_1}\gamma_1\left(\xi\right)d\xi \\ \tilde{x}_2\left(x_2\right) = \int_0^{x_2}\gamma_2\left(\xi\right)d\xi \\ \tilde{x}_3\left(x_3\right) = x_3 \end{cases}$$

(3)

where $\tilde{x}_1$, $\tilde{x}_2$ and $\tilde{x}_3$ denote the stretched coordinates. $\gamma_1\left(x_1\right)$, $\gamma_2\left(x_2\right)$ are non-zero, continuous, complex-valued coordinate stretching function.

2.3 Implementation of the governing equations into a commercial software package

The governing equation of the system can be written into a commercial finite element method package. In the package, the input formula for eigenvalue problems has the general expression as

$$\nabla\cdot\left(c\nabla\mathbf{u} + \alpha\mathbf{u} - r\right) - a\mathbf{u} - \beta\cdot\nabla\mathbf{u} + d_a\lambda\mathbf{u} - e_a\lambda^2\mathbf{u} = f$$

(4)

in which $\mathbf{u}$ represents the set of variables to be determined; the various coefficients from the input formula do not have any particular meaning except that they represent functions of the parameters of the problem investigated.

3. VALIDATIONS AND APPLICATIONS

In this section, one example case is shown to validate the developed model.

This example simulates Rayleigh surface wave propagation in a semi-infinite half space which is subjected to a depth-varying stress in near-surface area, and it was solved in the literature by perturbation method[3]. To model the propagation of the waves along an infinitely wide space, a narrow strip of the structure with periodic boundary condition (PBC) are used to represent continuity of displacements and stresses between the two edges, as shown in Fig. 1. The narrow strip of the structures is 1 mm in width, and the thickness of the space is set to be 400 mm, which is larger than one wavelength of Rayleigh wave. A 1-mm-thick PML is attached to the strip to simulate the semi-infinite half space.

A depth-varying static compressive stress having isotropic in-plane components $\sigma^S_{11} = \sigma^S_{33} = \sigma^S\left(x_2\right)$ and null vertical component $\sigma^S_{33} = 0$ is applied in the near-surface region. The amplitude of the static stress is scaled to material's bulk modulus $K = \lambda + \frac{2}{3}\mu$ and $H = 10\,\text{mm}$. In the simulation, four kinds of material, including aluminum, steel, the titanium alloy Ti-6426 and glass are chosen for being representative of a wide panel of industrial uses. All the material properties are given in Table I.

Table I. Material properties used in the calculations.

	ρ (g/cm^3)	λ	μ	*l*	*m*	*n* (GPa)
Ti	4.54	80.0	45.5	-201	-272	-356
Si	2.28	18.8	26.8	29	14.7	-26.8
Al	2.8	54.9	26.5	-252.2	-324.9	-351.1
Steel	7.8	115.8	79.9	-248	-623	-714

Figure 2 shows variation of velocity of the Rayleigh surface wave in different material subjected to the depth-varying stress. The velocity change is calculated by computing the difference between the velocity when the material is subjected to the applied stress and the velocity when the material is free of the applied stress. It can be seen that the results from the method developed in this paper agree very well with the solutions from perturbation method[3].

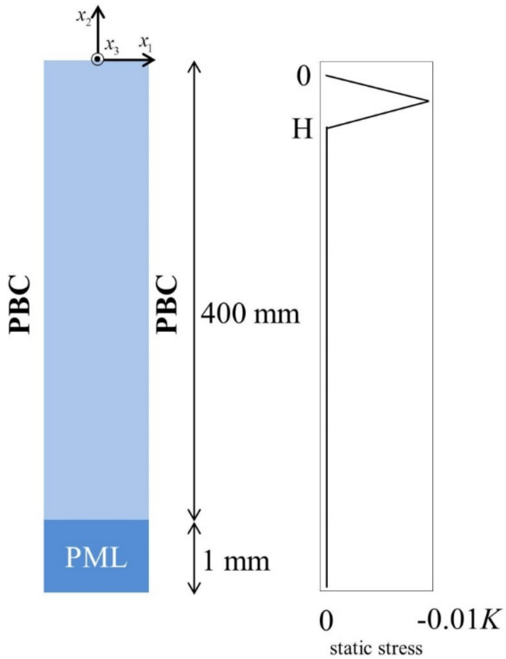

Figure 1: Schematic of the SAFE model for a semi-infinite half space subjected to a depth-varying static stress.

Some features can also be obtained from the results. It can be seen that the variation of velocity of the Rayleigh varies with frequencies and the variation of velocity becomes zero at very low and very high frequencies.

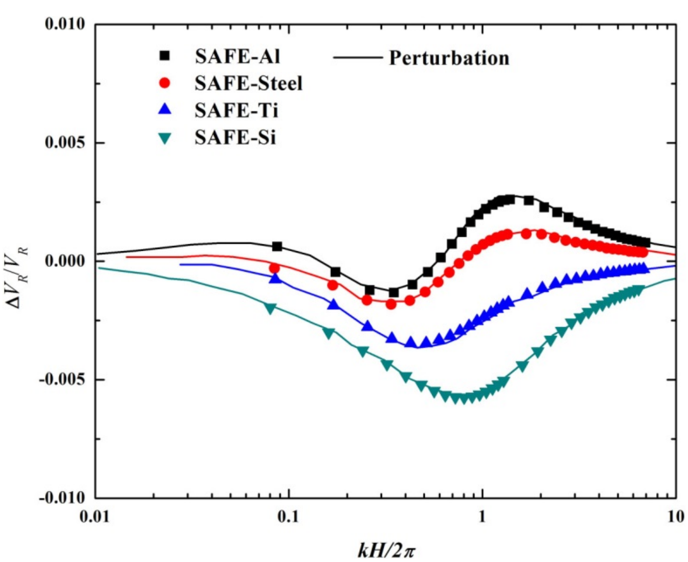

Figure 2: Variation of velocity of Rayleigh wave induced by a depth-varying compressive static stress profile localized in the near-surface region, in different materials.

4. CONCLUSION

A SAFE based guided wave model has been developed to study modal properties of guided waves in complex waveguides in a wide range of industries. The solutions from the model are compared with well-established solutions, showing excellent agreement. Our model provides a generic tool for the development of guided wave based techniques.

ACKNOWLEDGEMENTS

This work was supported by MOE AcRF Tier 1, RG99/17.

REFERENCES

[1] Fan, Z., Lowe, M. J. S., Castaings, M., Bacon, C., Torsional wave propagation along a waveguide of arbitrary cross section immersed in a perfect fluid, J. Acoust. Soc. Am. 124 (4) (2008) 2002-2010.

[2] Stroh, A. N., Steady state problems in anisotropic elasticity, J. Math. Phys. 41 (1962) 77-103.

[3] Mora, P., Spies, M., On the validity of several previously published perturbation formulas for the acoustoelastic effect on Rayleigh waves, Ultrasonics 91 (2019) 114-120.

[4] Bérenger, J. P., A perfectly matched layer for the absorption of electromagnetic waves, J. Comput. Phys. 114 (1994) 185-200.