

ON SIMULTANEOUS GENERATION OF SH₀ AND S₀ HARMONICS BY THE EXCITATION OF SH₀ NONLINEAR ULTRASONIC WAVE MODE ON ALUMINIUM PLATES: A CASE STUDY

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ABSTRACT

Advancements in Nonlinear Ultrasonics (NLU) and Guided waves what is commonly termed as nonlinear guided waves enables to combine the penetration power of guided waves for early state damage detection and enhanced structural monitoring capabilities over a fairly large area. This study aims at the generation of harmonics for special case of excitation of SH₀ mode on Aluminium sheet. Phase velocity matching criteria was ensured for the feasibility of harmonic generation. The simultaneous generation SH₀ mode and S₀ mode as even-harmonics was observed in numerical simulation and validation is performed with experiments. Each mode is travelling at different velocities so that they can be monitored separately for specific cases. An advantage of this generated S₀ mode is that it is non-dispersive in nature as it is generated from a SH₀ mode which is also non dispersive. As a result of this excitation we can study early state defects or micro-crack evaluation.

Keywords: nonlinear ultrasonics, nonlinear guided waves, higher harmonics generation

NOMENCLATURE

NLU	Nonlinear ultrasonics
SH ₀	Fundamental Shear Horizontal mode
S ₀	Fundamental Symmetric mode
FFT	Fast Fourier Transform

1. INTRODUCTION

Nonlinear ultrasonic guided wave is been of great interest to researchers in the past few decades owing to the advantages of guided waves on their range of inspection and NLU on the ability to inspect micro cracks and other precursors to structural failure. Higher harmonic generation from non-dispersive ultrasonic waves for material characterisation and nondestructive evaluation has been studied. M Deng [1], [2] through a series of article demonstrated the analytical and numerical investigation of harmonic generation in plates for Rayleigh Lamb and Shear Horizontal modes stated that second harmonics will be only generated in certain cases due to wave interaction. Later, based on these observations Lima and Hamilton [3] developed an analysis framework depicting the governing conditions for the cumulative harmonic generation, like synchronism (Phase

velocity matching and non-zero power flux from primary to secondary mode). Several articles [4]–[8] have used this framework to fill in many theoretical as well as numerical details.

NLU guided waves on plates with wave interaction approach in order to investigate multiple primary modes have been studied analytically, numerically and experimentally by Lissenden and his group through a series of works[5], [6], [8]. Peng Zuo et. al. [7] demonstrated that the amplitude of the attenuated second harmonic wave in immersed waveguides can keep constant with propagation distance, only if the primary wave is non-leaky. They excited the SH₀ mode and received S₀ mode at a particular frequency thickness. On the contrary we found that an SH₀ mode is also propagating along with S₀ mode. In this article we demonstrate the existence on simultaneous propagation of primary wave SH₀ mode and second harmonic S₀ and SH₀ mode via numerical simulations supported by experimental validations.

2. MATERIALS AND METHODS

Aluminium is considered by many researchers for nonlinear ultrasonic studies because of its polycrystalline nature which can demonstrate considerable material nonlinearity. We used aluminium sheet for this study as the third order elastic constants are well documented and the dispersion curves for same is plotted using Disperse [9]. FIGURE 1 shows the dispersion curve for 1 mm thick aluminium showing fundamental Rayleigh lamb waves and Shear Horizontal lamb waves overlaid with the second harmonic modes in dashed lines on double the frequency. Selection of mode pair is important as this will decide on the chances of generation of higher harmonics.

2.1 Mode pair Selection

Nonlinear ultrasonic wave propagation numerical simulations, theoretical formulation and solution of the second order harmonics on guided medium in isotropic plates are available. Mode selection is critical for the maximum interaction of wave and thus generate efficient harmonics. As per literature internal resonance criteria need to be satisfied for cumulative harmonic excitation, i.e.

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$$f_n^{surf} + f_n^{vol} \neq 0 \quad (\text{Non-zero power flux}) \quad (1)$$

$$k_m = 2k_n \quad (\text{Phase matching})$$

Where f_n^{surf} and f_n^{vol} denotes the power flux from the primary waves to secondary waves through the surface and volume of the waveguide respectively. Also k_m and k_n are the wave numbers of the secondary and primary wave modes. The phase matching condition ensures synchronism of the wavelengths of primary and second-order harmonic waves. If the internal resonance criteria are met for planar waves propagating in a lossless media, then the secondary waves will cumulatively increase until the wave interacting zone.

From the phase velocity dispersion curve shown in FIGURE 1, we can see the point where fundamental symmetric mode (s0) second harmonic wave mode meets primary shear horizontal mode (SH0) at 1.72 MHz. This satisfies the phase velocity matching criteria satisfying internal resonance criteria which is required for the optimum generation of higher harmonics. So we choose 1.72 as the excitation frequency for SH0 mode on aluminium plate of 1mm thick and hence our mode pair selected would be SH0-s0. It is also interesting to see that the same point there is a possibility of another mode pair of SH0-sh0, since shear horizontal mode have constant velocity throughout dispersion curve.

Density (Kg/m ³)	Lame's constants (GPa)		Murnaghan's constants (GPa)		
ρ	λ	μ	l	m	n
2700	51	26	-250	-333	-350

TABLE 1: ELASTIC CONSTANTS USED FOR ALUMINIUM FOR NUMERICAL SIMULATIONS

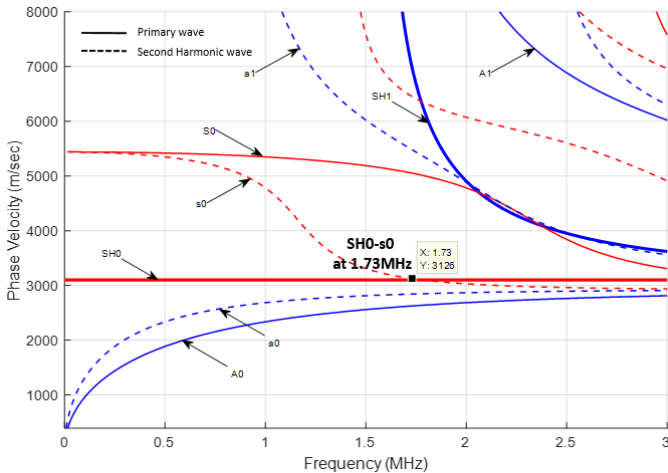


FIGURE 1: DISPERSION CURVE FOR THE 1 MM ALUMINIUM SHEET SHOWING PRIMARY MODES AND SECOND HARMONIC WAVE MODE IN DASHED LINES AND PLOTTED AS IF ON 2FD

2.2 Numerical Analysis

Finite element simulations are performed to explore the aspects associated with higher harmonic generation. Nonlinear ultrasonic wave propagation numerical simulations in this article are based on Murnaghan's NLU models and are performed using a commercially available Finite Element package COMSOL Multiphysics 5.3a using the hyper elastic module to perform nonlinear ultrasonic wave propagation study. Simulations carried out using Murnaghan's model available for Aluminium with elastic constants tabulated in TABLE 1.

A time domain 3D finite element model was developed based on the dispersion study for higher harmonic generation by phase velocity matching for SH0-s0 mode pair. A schematic of the FE model studied is shown in FIGURE 2. A tone burst of 40 cycles 1.72 MHz frequency was used with a factor of 1e-7 as load for excitation as displacement boundary condition. SH0 mode edge excitation is given at the end face of a rectangular strip of 1mm thickness. The maximum element size for the mesh was 0.1 mm ($\lambda/8$) and time step was 0.02 μ s to ensure stable numeric convergence. Periodic boundary condition is given at both lateral faces to realize the plate continuity and minimize the FE model size for faster execution. The approach for excitation of SH0 excitation on Aluminium plates mention by Mostafa [6] and Zuo [7] is followed here.



FIGURE 2: SCHEMATIC OF GEOMETRY FOR FE SIMULATION

3. RESULTS AND DISCUSSION

In plane (excitation direction) and Out of plane displacement at different locations along the direction of SH0 wave propagation were monitored. A-scans and corresponding frequency spectrum for monitoring points placed at regular intervals are shown in FIGURE 3. Images on the top in blue colour is in-plane displacement corresponding to SH0 mode and Images on the bottom in red colour is out of plane displacement corresponding to S0 mode as per selected mode pair. From the frequency domain plots is to be noticed that SH0 mode is having its fundamental frequency (f_0) and next harmonic is at 3 times frequency ($3f_0$). Whereas S0 mode group is showing frequencies at even harmonics ($2f_0$ and $4f_0$).

In the bottom set of figures, you can see two packets moving at different velocities leading to further the investigation on the behaviour.

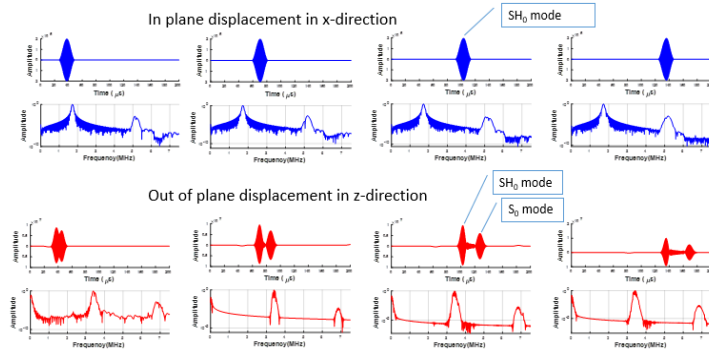


FIGURE 3: A-SCANS AND CORRESPONDING FREQUENCY SPECTRUM FROM THE MONITORS ALONG THE DIRECTION OF PROPAGATION SHOWING INPLANE DISPLACEMENTS (IN BLUE COLOUR) AND OUT OF PLANE DISPLACEMENTS (IN RED COLOUR)

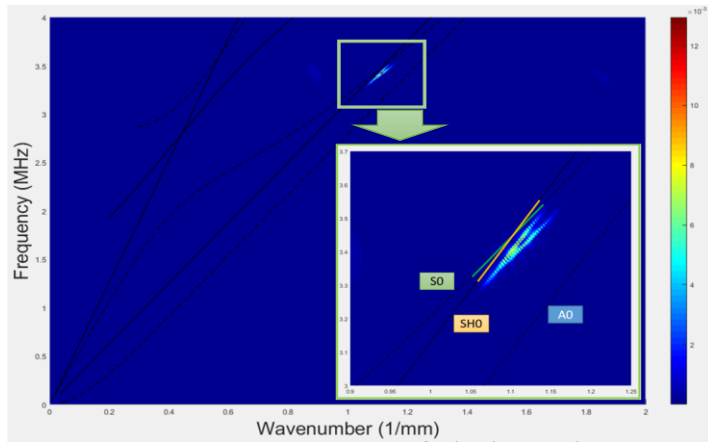


FIGURE 4: 2DFFT FOR THE RECEIVED SIGNAL SHOWING THE PRESENCE OF BOTH S0 MODE AND SH0 MODE FROM THE NUMERICAL SIMULATIONS.

As further investigation 2D FFT of the out of plane displacements were plotted and dispersion curve was overlaid over it to identify the wave modes as shown in FIGURE 4. In the zoomed and cropped image in the inset we can clearly see that two separate wave modes are present whose slope corresponds to SH0 mode and s0 mode at second harmonic frequency (2f0). Which makes us think of the possibility of another mode pair (SH0-sh0) at same frequency, since shear horizontal mode have same phase velocity throughout the frequency range.

Experimental validation details are not covered here, as they are still in progress and will be presenting at the conference.

4. CONCLUSION

Thus in this paper we conclude that two wave modes (sh₀ and s₀) corresponding to SH0-s₀ and SH0-sh₀ mode pairs propagate with second harmonics along the propagation direction. This special case can give extra flexibility in analyzing defects with different characteristics to which they responds to. Early state detection of fine defects or micro cracks sensitive to the wave modes based on the mode shape can provide a leading

edge above the other second harmonic generation which deals with one dominant wave mode.

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