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SIMULTANEOUS CHARACTERIZATION OF ORTHOTROPIC ELASTIC AND ATTENUATION PROPERTIES IN UNIDIRECTIONAL POLYMER MATRIX COMPOSITE PANELS

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ABSTRACT

Ongoing efforts related to the modeling of the ultrasonic inspection of impact damage in polymer matrix composite materials has necessitated the accurate measurement of elastic and attenuation properties in the material. Techniques for these measurements are well-documented in literature; however, some of the details related to integration of a forward modeling and inversion process with an experimental measurement are somewhat lacking in literature. This paper discusses progress toward development of a complete and fully integrated technique that quickly and simultaneously characterizes orthotropic elastic and attenuation properties in polymer matrix composite panels. Code is being developed, in Python, to implement the forward model and inverse process with the intent of release to the community as open-source.

Keywords: ultrasound, composites, elastic moduli

1. INTRODUCTION

In support of ongoing efforts to model the ultrasonic inspection of impact damage in polymer matrix composite (PMC) materials [1-4], there is a need for accurate characterization of both elastic properties and attenuation properties for an individual layer of the layup. Numerous techniques are available to measure these properties for general material systems, as well as techniques that are applicable to the specific challenges associated with PMC panels. However, much of the documentation in literature describes small pieces of the puzzle, with less attention paid to integration into a complete measurement process. This work details a complete measurement process, including experimental measurement, forward model, and inverse process, to determine the full set of independent orthotropic elastic and attenuation properties in a PMC panel.

2. THEORY

The fundamental measurement scheme used involves measuring the response of the panel to ultrasonic bulk plane waves introduced at a variety of different incident angles and orientations while under water immersion. We can then apply Christoffel's equation which is given by

$$\left(c_{ijlm} n_j n_l - \rho V^2 \delta_{im}\right) p_m = 0 \tag{1}$$

where c_{ijlm} is the elastic tensor, n_j is the jth component of the unit vector in the direction of wave propagation, ρ is density, V is the phase velocity, p_m is the mth component of the polarization vector, and δ_{im} is the Kronecker delta function. From Equation 1, we readily see a relationship between wave speed and orientation of the wave passing through a sample. To have meaningful solutions, the determinant of the portion of Equation 1 inside the brackets must be zero. This eigenvalue problem has three solutions in V^2, one associated with each of the wave modes present in the sample [5].

Equation 1 provides an analytic forward model for wave speed through the sample at a particular orientation. Wave speed can be readily measured experimentally. At varied orientations, different components of c_{ijlm} will have varying influence on the ultimate wave speed. Therefore, we must experimentally measure wave speed at a sufficiently large number of orientations. We can then determine c_{ijlm} through a nonlinear optimization routine.

However, this will not directly provide information on attenuation properties. Several techniques exist to compute theoretical reflection and transmission coefficients through a material to therefore determine the expected attenuation through the sample given a known incident wave field. A particularly well-suited technique is the stiffness matrix approach of Wang and Rokhlin [6].

A viscoelastic model is utilized, where c_{ijlm} is complex with a real component associated with elastic behavior and an imaginary component associated with attenuation behavior. Using the coordinate system in Figure 1, we must consider waves travelling both up and down through the thickness of the panel. The stresses and displacements at the top and bottom of the panel are related by the equation

$$\begin{bmatrix}\boldsymbol{\sigma}_T\\ \boldsymbol{\sigma}_B\end{bmatrix} = \begin{bmatrix}\boldsymbol{D}^- & \boldsymbol{D}^+\boldsymbol{H}^+\\ \boldsymbol{D}^-\boldsymbol{H}^- & \boldsymbol{D}^+\end{bmatrix}\begin{bmatrix}\boldsymbol{P}^- & \boldsymbol{P}^+\boldsymbol{H}^+\\ \boldsymbol{P}^-\boldsymbol{H}^- & \boldsymbol{P}^+\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{u}_T\\ \boldsymbol{u}_B\end{bmatrix} = \boldsymbol{K}\begin{bmatrix}\boldsymbol{u}_T\\ \boldsymbol{u}_B\end{bmatrix} \tag{2}$$

where σ_T, σ_B, u_T, and u_B are the stresses and displacements on the top and bottom of the panel, + superscripts represent wave modes travelling upward, - superscripts represent wave modes travelling downward, and the remaining terms are defined as follows:

$$\mathbf{D}^{\pm} = [\mathbf{d}^{\pm 1}, \mathbf{d}^{\pm 2}, \mathbf{d}^{\pm 3}] \tag{3}$$

$$\mathrm{d}_q^{\pm n} = \mathrm{i}c_{q3tm}\mathrm{k}_m^{\pm n}\mathrm{p}_t^{\pm n} \tag{4}$$

$$\mathbf{P}^{\pm} = [\mathbf{p}^{\pm 1}, \mathbf{p}^{\pm 2}, \mathbf{p}^{\pm 3}] \tag{5}$$

$$\mathbf{H}^{\pm} = \begin{bmatrix} e^{\mathrm{i}k_z^{\pm 1}h} & 0 & 0\\ 0 & e^{\mathrm{i}k_z^{\pm 2}h} & 0\\ 0 & 0 & e^{\mathrm{i}k_z^{\pm 3}h}\end{bmatrix} \tag{6}$$

where $\mathbf{d}^{\pm n}$ represents a traction vector for the nth wave mode, $\mathrm{k}_m^{\pm n}$ is the mth wave number component for the nth wave mode, $\mathbf{p}^{\pm n}$ is the polarization vector for the nth wave mode, and h is the panel thickness. $\boldsymbol{K}$ is named in [6] as the stiffness matrix, and its inverse $\boldsymbol{K}^{-1} = \boldsymbol{S}$ is named the compliance matrix. It is important to note that these matrices are as defined in Equation 2 and should not be confused with the stiffness tensor of Hooke's law. Both matrices are 6×6 and can be split up into four 3×3 submatrices for convenience [6].

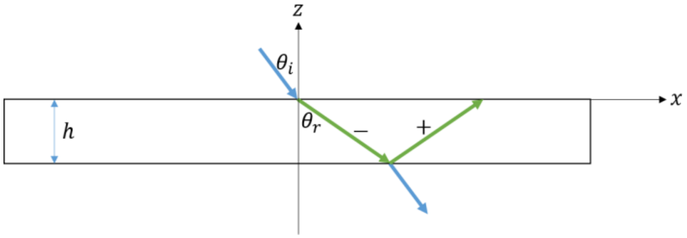

FIGURE 1: COORDINATE SYSTEM FOR FORWARD MODEL

Using the compliance matrix, we can compute reflection and transmission coefficients as

$$T = -\frac{2\Lambda \boldsymbol{S}_{21}^{33}}{(\boldsymbol{S}_{11}^{33} + \Lambda)(\boldsymbol{S}_{22}^{33} - \Lambda) - \boldsymbol{S}_{21}^{33}\boldsymbol{S}_{12}^{33}} \tag{7}$$

$$R = -\frac{(\boldsymbol{S}_{11}^{33} - \Lambda)(\boldsymbol{S}_{22}^{33} - \Lambda) - \boldsymbol{S}_{21}^{33}\boldsymbol{S}_{12}^{33}}{(\boldsymbol{S}_{11}^{33} + \Lambda)(\boldsymbol{S}_{22}^{33} - \Lambda) - \boldsymbol{S}_{21}^{33}\boldsymbol{S}_{12}^{33}} \tag{8}$$

$$\Lambda = -\frac{\cos\theta_i}{i\omega\rho_f V_f} \tag{9}$$

where $\boldsymbol{S}_{xy}^{33}$ is the (3,3) component of the (x, y) submatrix of $\boldsymbol{S}$, θ_i is the incident angle of the wave, ω is the angular frequency, ρ_f is the density of the water, and V_f is the speed of sound in water [6].

T and R are computed as a function of frequency. They can then be multiplied by the frequency domain signal of a waveform incident on the top surface of the panel. The inverse Fourier transform of the result will give the time domain reflection and transmission response of the panel. An example forward model run result compared with experimental results is shown in Figure 2 for a normal incidence case in an IM7/977-3 unidirectional layup. The real experimental results come from a 4.66 mm thick $[0]_{36}$ layup constructed from prepreg material.

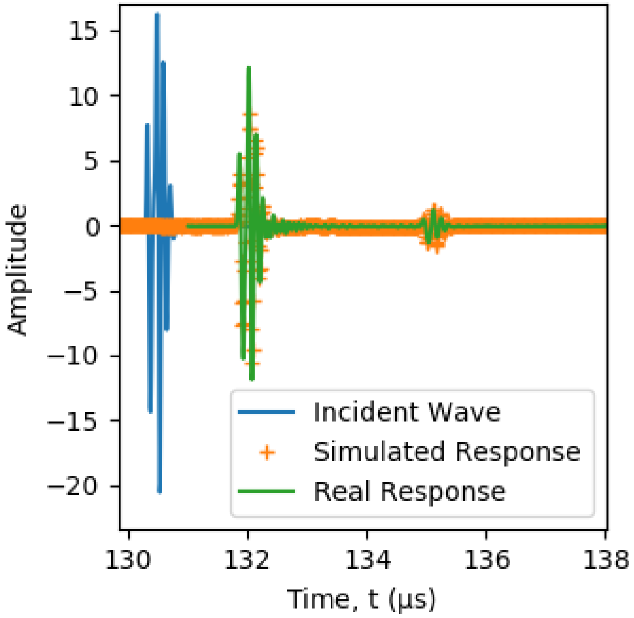

FIGURE 2: SAMPLE OF FORWARD MODEL RUN RESULTS

Both forward model and inverse optimization are implemented in Python. The inverse process is accomplished by means of a nonlinear optimization. Additional details on improvements to the forward model and inverse process will be presented. Additionally, it is the author's intent to publish the code developed as open source and freely available.

3. EXPERIMENTAL MEASUREMENT SETUP

The experimental data is collected using a double through-transmission pulse-echo technique [7]. A photo of the experimental setup can be seen in Figure 3. A reflector plate and the ultrasonic transducer, in this case a 10 MHz 12.7 mm diameter unfocused probe, are held fixed and in alignment with each other. A reference measurement is taken without a sample installed. This becomes the incident wave field seen in Figure 2. The sample is then installed on a fixture to allow rotation of the sample through different orientations, and the platform it is attached to below allows automated rotation through various incident angles.

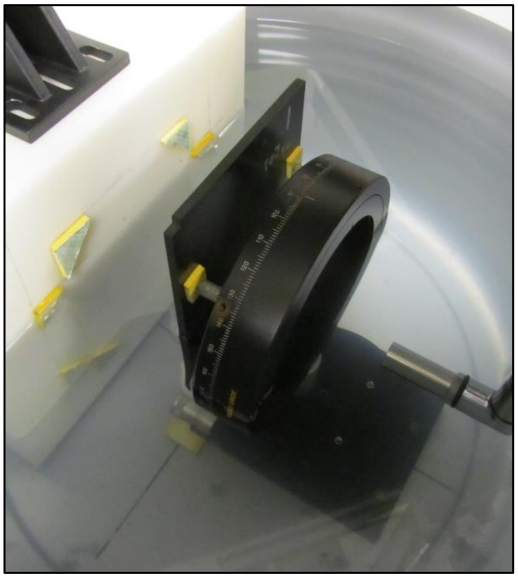

FIGURE 3: EXPERIMENTAL MEASUREMENT APPARATUS

Full measurement results and improvements to the process will be presented.

4. CONCLUSIONS

A complete start-to-finish process has been implemented for measuring fully orthotropic elastic and attenuation properties in unidirectional PMC panels. A set of code has been developed to calculate the forward model and for the inverse optimization process that will be published as open source. Experimental measurements were performed on an IM7/977-3 unidirectional panel with reasonable agreement with expected results.

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