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INVESTIGATION OF THE CIRCUMFERENTIAL GUIDED WAVES PROPAGATION IN FUNCTIONALLY GRADED HOLLOW CYLINDERS BY STATE VECTOR FORMALISM AND LEGENDRE POLYNOMIAL HYBRID METHOD

Jie Gao, Cunfu He, Yan Lyu [1], Guorong Song, Bin Wu
College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology,
Beijing 100124, China

ABSTRACT

By introducing the state matrix into the orthogonal polynomial approach, this paper investigates the circumferential guided waves propagation in functionally graded material (FGM) hollow cylinders without discretizing into homogeneous multi-layered model. As an alternative new approach, which avoids the complicated integral operation in the conventional orthogonal polynomial series method in virtue of the orthogonal completeness and strong recursive property of Legendre polynomial series. Comparison with available results from the Lamb wave dispersion curves of the equivalent plate, the feasibility and validity of the proposed method is verified effectively. In order to demonstrate the robustness and versatility of the state vector formalism and Legendre polynomial hybrid method, the iron based alumina FGM with different graded structure are researched, and influence of the radius-to-thickness ratio and graded field on the dispersion characteristic are discussed, respectively.

Keywords: Legendre polynomial method; circumferential guided wave; dispersion curves

NOMENCLATURE

u	displacement
σ	stress
ε	strain
c_{ijkl}	elastic constants (i,j,k,l=1,2,3)

1. INTRODUCTION

Due to the strong designability, functionally graded materials (FGM) are widely used in the fields of materials are used to energy conversion, mechanical engineering, and nuclear applications [1]. In order to improve the service life of functionally graded materials and reduce the application cost ultrasonic guided waves technique has received a great deal of attention in non-destructive evaluation field.

Moreover, mathematical modeling of the axial propagation of guided waves in hollow cylinder has been reported extensively. For wave propagation in the circumferential direction, the pioneering work on wave propagation characteristics of one or multiple curved surfaces has been analyzed by Grace and Goodman [2], Liu and Qu [3] and so on. In addition, the Disperse commercial simulation software [4], developed by the NDT research team at imperial college London, which can be obtained the dispersion curves of circumferential guided waves for isotropic cylinders. And based on the Fourier series expansion method, Towfighi [5] solved the elastic wave propagation problem in anisotropic cylindrical plates in the circumferential direction. For more than forty years, the orthogonal polynomial series approach has been proposed to solve wave problem [6].

In this research, we present a hybrid method for studying the circumferential guided wave for FGM hollow cylinder without discretizing into homogeneous multi-layered model. At the same time, the influence rule of graded field and radius-to-thickness ratio on dispersion propagation is revealed.

2. MATERIALS AND METHODS

Consider a FGM hollow cylinder with varying material properties with regard to thickness h ($0<h<b-a$), as shown in Fig.1, and the circumferential guided waves propagate along the θ direction. In the cylindrical coordinate system(r, θ, z), we define the radius-to-thickness ratio as η, $\eta=b/(b-a)$.

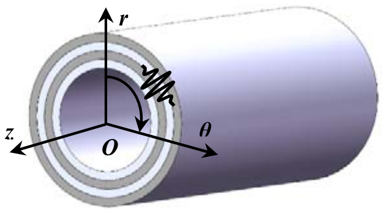

FIGURE 1: THE GEOMETRY OF FGM HOLLOW CYLINDERS

We assumed that the FGM plates are made of two different materials, which are denoted M1, M2, respectively. The properties of internal materials are gradually changed from M1

[1] Contact author: lvyan@bjut.edu.cn

to M2. Meanwhile, we choose the Voigt-type model [7] to calculate the material parameters of FGM pipe, which can be expressed as:

$$C_{IJ}\left(r\right)=C_{IJ}^{2}+\left(C_{IJ}^{1}-C_{IJ}^{2}\right)\left(1-\frac{r-a}{h}\right)^{p}$$
$$\rho\left(r\right)=\rho^{2}+\left(\rho^{1}-\rho^{2}\right)\left(1-\frac{r-a}{h}\right)^{p}\quad\left(0\le p\le\infty\right)\tag{1}$$

To transform the fundamental equation of the circumferential guide wave in the cylinder into the state matrix forms, the displacement and stress expression can be arranged as the following:

$$\mathbf{u}=\left[u_{r}\quad u_{\theta}\quad u_{z}\right]^{T}e^{i(k\theta-\omega t)}$$
$$\boldsymbol{\tau}_{i}=\left[\sigma_{ir}\quad\sigma_{i\theta}\quad\sigma_{iz}\right]^{T}e^{i(k\theta-\omega t)}\tag{2}$$

Then, the governing equation and the stress vector in term of the displacement vector can be rewritten respectively as:

$$\frac{\partial r}{\partial r}\boldsymbol{\tau}_{r}=r\rho\left(r\right)\omega^{2}[I]\mathbf{u}-\left(\frac{\partial}{\partial\theta}+M\right)\boldsymbol{\tau}_{\theta}-\frac{\partial}{\partial z}\boldsymbol{\tau}_{z}\tag{3}$$

$$\boldsymbol{\tau}_{i}=\frac{\partial}{\partial r}\left[D_{i1}\right]\mathbf{u}+\left(\frac{\partial}{\partial\theta}+M\right)\frac{1}{r}\left[D_{i2}\right]\mathbf{u}+\frac{\partial}{\partial z}\left[D_{i3}\right]\mathbf{u}\tag{4}$$

where $[D_{ij}]$ are related to elastic constants matrix, and M is represent as follow:

$$M=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}\tag{5}$$

Based on the Eqs. (1-4), the dispersion characteristic equation of FGM hollow cylinder can be yielded:

$$\begin{aligned}&-k^{2}\left(\left[D_{22}\right]\right)u+ikr\left(\left[D_{12}\right]+\left[D_{21}\right]\right)\frac{\partial u}{\partial r}+ik\left(\left[D_{22}\right]M+M\left[D_{22}\right]\right)u\\&+\frac{\partial u}{\partial r}r\left(\left[D_{11}\right]+\left[D_{12}\right]M+\left[D_{21}\right]M\right)+r^{2}\frac{\partial^{2}u}{\partial r^{2}}\left[D_{11}\right]\\&+M\left[D_{22}\right]Mu+r^{2}\rho\omega^{2}[I]u+r^{2}\frac{\partial u}{\partial r}\left[D_{11}\right]'+r\left[D_{12}\right]'\left(ik+M\right)u=0\end{aligned}\tag{6}$$

Next, according to the Galerkin calculation principle, we adopt the Legendre polynomial series as an orthogonal basis function in order to realize the superposition fitting of displacement field. And the particle displacement of sound field is expressed as:

$$u=\sum_{n=0}^{N-1}U_{n}P_{n}(\chi)\tag{7}$$

where the $P_n(\chi)$ is the n^{th} order Legendre polynomial, U_n represents the expansion coefficients of the amplitudes of the three displacement components. And N is the cut-off order of the selected Legendre polynomial series. However, the working range of Legendre polynomial is [-1, 1], it is necessary to move $r \in [a, b]$ into the above range, as:

$$\chi=\ell(r-R),\quad\ell=2/h,\quad R=(a+b)/2\tag{8}$$

To simplify the computation, the volume fraction of graded structure can be fitted with the univariate nonlinear regression by using the "Curve Fitting" procedure from MATLAB software. Meanwhile, by virtue of the recursive property of Legendre polynomials, the analytic expression of the corresponding linear operator can be obtained significantly. Moreover, submitting the Eqs. (7, 8) into the dispersion equation, multiplying both sides of Eq. (6) by $P_m(\chi)$, in which the m from zero to cut-off order M, and integrate χ from −1 to 1.And taking advantage of the orthogonality of Legendre polynomials, a sufficient number of linear independent equations can be achieved, and combing with the boundary condition ($\boldsymbol{\sigma}_r$=0), which can be express as the matrix equation:

$$\left(\begin{bmatrix}\boldsymbol{\Psi}_{3N\times3N}&-I\\-\boldsymbol{\Gamma}_{3N\times3N}&0\end{bmatrix}-\frac{j}{\xi}\begin{bmatrix}\mathbf{Z}_{3N\times3N}&0\\0&I\end{bmatrix}\right)\begin{bmatrix}\mathbf{D}_{3N\times1}\\\mathbf{R}_{3N\times1}\end{bmatrix}=0\tag{9}$$

Finally, by solving the eigenvalues and eigenvectors of the Eq. (9), the dispersion characteristic and displacement, stress distribution curves along the thickness direction can be derived simultaneously.

3. RESULTS AND DISCUSSION

According to the foregoing formulations in above section 2, the relevant computer programs have been developed in terms of MATLAB software, permitting to compute the dispersion curves of circumferential guided waves in FGM hollow cylinder. In this research, the Fe-Al_2O_3 FGM hollow cylinder are selected as the research objects, which the mechanical properties are collected in Table 1. In order to verify the validity of the programming and theoretical formulas, we calculate the dispersion curves of circumferential guided waves for FGM pipe with sinusoidal graded field $V_1(r)=\sin(0.5\pi(1-r/h))$, where the η set to approximate infinity, and compared to corresponding equal thickness FGM plate, which is obtained from our previous work. And the dispersion curves are shown in Fig. 2, where the black dotted line are the hollow cylinder's results, and the red dotted line are results from plate. It clearly indicate that they both match well to each other.

Table 1 Mechanical properties of materials used [8].

Materials	ρ(kg/m^3)	C_{11}(GPa)	C_{12}(GPa)	C_{44}(GPa)
Fe	7870	218.5	63.6	77.5
Al_2O_3	3900	388.8	85.5	140.0

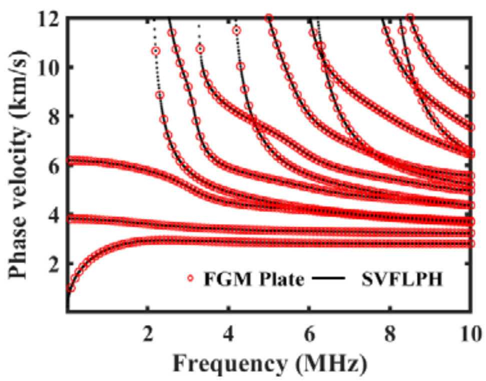

FIGURE.2 COMPARISON OF THE DISPERSION CURVES FOR IRON BASED ALUMINA BY PIPE AND PLATE

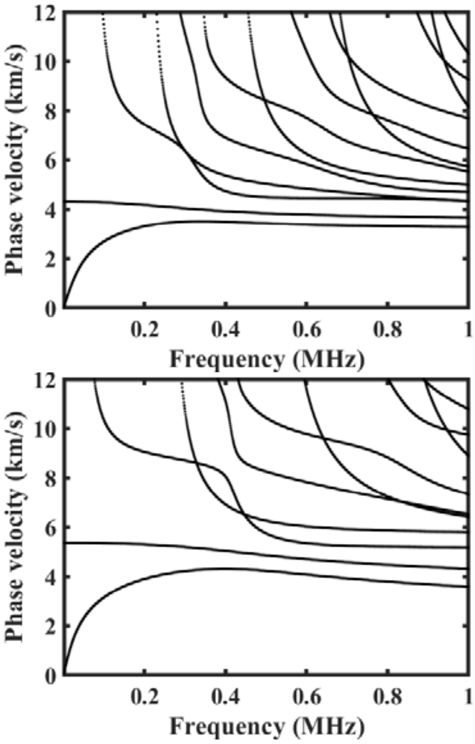

FIGURE.3 THE DISPERSION CURVES FOR IRON BASED ALUMINA FGM PIPE: p=1, p=5.

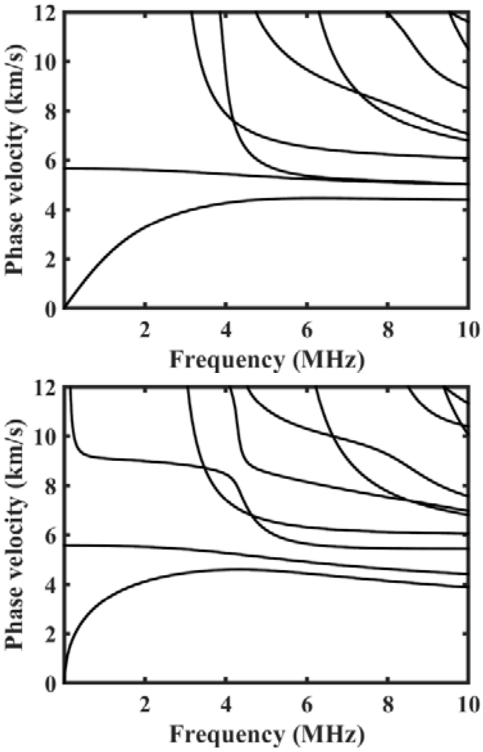

FIGURE.4 THE DISPERSION CURVES FOR IRON BASED ALUMINA FGM PIPE: η=4, η=100

In order to illustrate the effect of graded field and radius-to-thickness ratio on the dispersion curves of circumferential guided waves propagation in Iron based alumina FGM hollow cylinders, we calculate the corresponding dispersion curves in the Fe-Al2O3 FGM pipe when p=1,5 and η=4,100, respectively, as shown in Figs. (3, 4).In former case, the geometrical features are a=60mm, b=70mm; the latter are p=8, h=1mm, respectively. It can be clearly seen that, in Fig.3, as the increase of gradient index p, elastic mode number decrease gradually; in contrary, the elastic mode increase with the radius-to-thickness ratio η in the range of 0MHz*mm-10MHz*mm. That is to say, it is apparent that both the gradient structure and the radius-to-thickness ratio affect the dispersion characteristic of circumferential guided waves propagation in FGM hollow cylinders obviously.

4. CONCLUSION

This paper presents an analytic approach based on state matrix and Legendre polynomial expansions method for the dispersion characteristic of the circumferential guided waves for functionally graded material (FGM) hollow cylinders without discretizing the gradient structure into homogeneous multi-layered model. The correctness and feasibility are verified by the numerical result comparison with available compute results from the FGM plate with corresponding distribution of mechanical properties, which is equal thickness to pipe. A perfect agreement was found between both sets of dispersion curves. In addition, we investigated the dispersion curves of circumferential guided waves propagation in Iron based alumina FGM hollow cylinder with different graded profile and the radius-to-thickness ratio, respectively. And the relationship between the gradient distribution, radius-to-thickness ratio and propagation characteristics can be established effectively by state vector formalism and Legendre polynomial hybrid method. Moreover, future work will be continued to extend the proposed method to deal with the wave problem of FGM structure with multiple physical field coupling.

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