

**EDDY CURRENT MEASUREMENT OF LAYERS THICKNESSES BASED ON REFLECTION  
COEFFICIENT'S SPECTRUM ANALYSIS**

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## ABSTRACT

The eddy current measurement signals of layered structures are analyzed in terms of transmission and reflection between the layers, so that the contributions from each layer can be clarified. The thicknesses are characterized based on reflection coefficient's spectrum analysis.

Keywords: eddy current measurement, reflection, spectrum, layered structure

## NOMENCLATURE

$\lambda_0$	wave number
$\omega$	angular frequency
$\mu$	magnetic permeability
$\sigma$	conductivity
$R(\lambda_0)$	general reflection coefficient

## 1. INTRODUCTION

Eddy current measurement is one of the most extensively studied electromagnetic measurement techniques to characterize multilayered structures. The signal of eddy current measurement, namely, the impedance change of probe, manifests the interaction between probe and test object; it depends on frequency, the probe's geometry and setup, the geometry and electromagnetic characteristics of the test object. Because of the interaction and multi-interference of the electromagnetic waves, the signals of a multilayered structure are integral of all the layers. In order to characterize a particular layer, the influence from the other layers should be excluded and the signal of the specific layer should be extracted. To separate signals or to construct characteristic features that are sensitive to the layer of interest but insensitive to other layers hold the key to multilayered structures characterization.

Previous study on pulsed eddy current testing (PECT) of pipe line covered by insulation and cover sheet showed that the PECT signals could be decoupled to some extent in the time domain [1]. Decay rate of the time-varying signals is insensitive to the variation of probe liftoff and inclination and thus is a characteristic feature for pipe's thickness evaluation. Swept-frequency eddy current testing (SFECT) to measure the thickness of a conductive plate or a non-conductive coating on it [2, 3] showed that the thickness of the plate or the coating could be evaluated even without knowing the conductivity of the test object. SFECT on air-gap-separated dual-layered structure [4] showed that the top layer and the lower layers could be evaluated respectively using high frequency and low frequency signals. The differential operation [4] showed that the differential of signals in the frequency series is almost invariant to the change of air gap.

Nonetheless, there are many unsolved issues in the eddy current measurement of layered structures that the layers' electromagnetic properties are generally different. First of all, we need to know whether the signal of a particular layer could be extracted. We also need to deal with fluctuated or uncertain factors such as the variation of probe setup, the uncertainty of electromagnetic properties and etc.

In this work, we derive analytical solution to calculate eddy current measurement signals in terms of the transmission and reflection of electromagnetic waves between the layers, and characterize the layers based on spectrum analysis of the reflection coefficients.

## 2. THEORETICAL STUDY IN TERMS OF TRANSMISSION AND REFLECTION

Multilayered structures are usually modelled by planar layers in theoretical analysis. Consider a self-induction coil shown in Fig. 1. The cylindrical air-cored coil (inner and outer radius  $r_1$  and  $r_2$ , thickness  $H$ ) carrying current of angular frequency  $\omega$  is placed with liftoff  $l$  above the test object. The change in coil impedance due to eddy current induction in the conductive object can be calculated by the following equation [2, 3]

$$\Delta Z(\omega) = \Delta R + j\omega\Delta L = j2\pi\omega\mu_0 n_{cd}^2 \int_0^\infty \frac{\chi^2(\lambda_0 r_1, \lambda_0 r_2)}{\lambda_0^6} (e^{-\lambda_0 l} - e^{-\lambda_0(l+H)})^2 R(\lambda_0) d\lambda_0, \quad (1)$$

where  $\mu_0$  is the magnetic permeability of free space,  $n_{cd}$  the turn density of the coil.  $\lambda_0$ , the integral parameter of the Bessel function, is also considered as wavenumber [5].  $R(\lambda_0)$  is the reflection coefficient only relevant to the test object [1,2]. Denoting the term  $\frac{\chi^2(\lambda_0 r_1, \lambda_0 r_2)}{\lambda_0^6} (e^{-\lambda_0 l} - e^{-\lambda_0(l+H)})^2$  in the integrand of Eq. (1) as shape function  $S$ ,

$$S(\lambda_0) = \frac{\chi^2(\lambda_0 r_1, \lambda_0 r_2)}{\lambda_0^6} e^{-2\lambda_0 l} (1 - e^{-\lambda_0 H})^2, \quad (2)$$

Eq. (1) is rewritten as

$$\Delta Z(\omega) = j2\pi\omega\mu_0 n_{cd}^2 \int_0^\infty S(\lambda_0) R(\lambda_0) d\lambda_0, \quad (3)$$

indicating that the EM signal is decided by the shape function  $S(\lambda_0)$  and the reflection coefficient  $R(\lambda_0)$ .

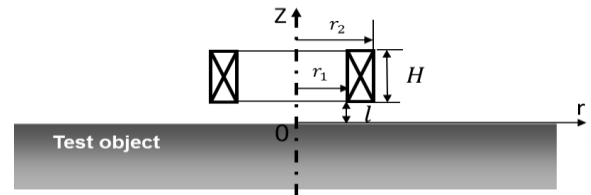


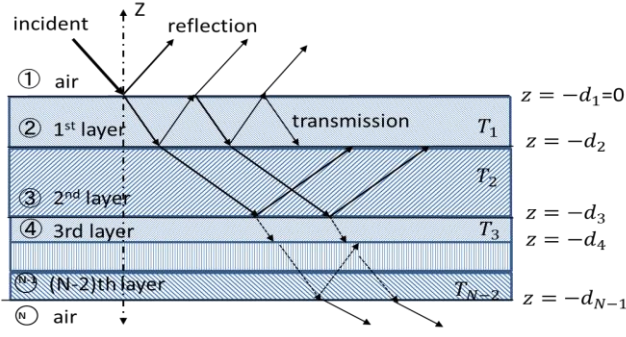
FIGURE 1: CONFIGURATION OF EDDY CURRENT MEASUREMENT USING A SELF-INDUCTION COIL

The eddy current measurement of layered structure can be interpreted as the transmission and reflection of waves between layers: waves generated by the excitation coil placed in region 1 incident to the layered media, transmit and reflect in the layers, reflect back to region 1 and received by pickup coil.

Assuming that each layer in Fig. 2 are homogenous, isotropic with uniform thickness and permeability  $\mu$ , conductivity  $\sigma$ . The general reflection coefficients between the  $i^{\text{th}}$  and the  $(i+1)^{\text{th}}$  regions is [4],

$$\tilde{R}_{i,i+1} = \frac{R_{i,i+1} + \tilde{R}_{i+1,i+2} e^{-2k_{i+1}(d_{i+1}-d_i)}}{1 + R_{i,i+1} \tilde{R}_{i+1,i+2} e^{-2k_{i+1}(d_{i+1}-d_i)}}, \quad (4)$$

where



**FIGURE 2:** TRANSMISSION AND REFLECTION OF WAVES BETWEEN LAYERS.

$$R_{i,i+1} = \frac{\mu_{i+1}k_i - \mu_i k_{i+1}}{\mu_{i+1}k_i + \mu_i k_{i+1}} \quad (5)$$

is the reflection coefficient at the interface of the  $i^{\text{th}}$  and the  $(i+1)^{\text{th}}$  regions. The  $\tilde{R}_{1,2}$  standing for the general reflection between regions 1 and 2 is equivalent to the  $R(\lambda_0)$  in Eqs. (1) and (3). Moreover, because there is no reflection between the  $N^{\text{th}}$  and the hypothetical  $(N+1)^{\text{th}}$  regions,  $\tilde{R}_{N,N+1} = 0$ .  $\tilde{R}_{1,2}$  can be attained recursively.

Hereafter are some typical situations.

### ① Conductive half space

Conductive half space is modelled by two regions: region 1 is the upper half space of air and region 2 is infinitely thick conductive media. Because of no reflection in region 2, the general reflect coefficient  $\tilde{R}_{1,2}$  is equal to the reflection coefficient  $R_{1,2}$ . By the way, in region 1,  $\sigma_1 = 0$ ,  $\mu_1 = \mu_0$ , so that  $k_1 = \lambda_0$  therefore

$$\tilde{R}_{1,2} = R_{1,2} = \frac{\mu_2 k_1 - \mu_1 k_2}{\mu_2 k_1 + \mu_1 k_2} = \frac{\mu_2 \lambda_0 - \mu_0 k_2}{\mu_2 \lambda_0 + \mu_0 k_2}, \quad (6)$$

where  $k_2 = \sqrt{\lambda_0^2 + j\omega\mu_2\sigma_2}$ . Eq. (6) shows that the amplitude of  $\tilde{R}_{1,2}$  depends on the electromagnetic properties of the media in region 2.

### ② A slab of limited thickness

The measurement of a limited thick slab can be modelled by three regions, where regions 1 and 3 are air, region 2 is the  $d_2 - d_1 = T_1$  thick conductive slab. The general reflection coefficient expressed by

$$\tilde{R}_{1,2} = \frac{R_{1,2} + \tilde{R}_{2,3} e^{-2k_2(d_2-d_1)}}{1 + R_{1,2} \tilde{R}_{2,3} e^{-2k_2(d_2-d_1)}} \quad (7)$$

shows that  $\tilde{R}_{1,2}$  changes with slab thickness.

For a single plate,  $\tilde{R}_{2,3} = R_{2,3} = -R_{1,2}$ . By substituting  $R_{1,2}$  and  $\tilde{R}_{2,3}$  into Eq. (7), we have the following expression

$$\tilde{R}_{1,2} = \frac{R_{1,2} - R_{1,2} e^{-2k_2 T_1}}{1 - R_{1,2}^2 e^{-2k_2 T_1}}. \quad (8)$$

Because the  $R_{1,2}^2 e^{-2k_2 T_1}$  in the dominator is smaller than 1,  $\tilde{R}_{1,2}$  is expanded to

$$\begin{aligned} \tilde{R}_{1,2} &= R_{1,2} (1 - e^{-2k_2 T_1}) (1 + R_{1,2}^2 e^{-2k_2 T_1} + R_{1,2}^4 e^{-4k_2 T_1} + \dots) \\ &= R_{1,2} - R_{1,2} e^{-2k_2 T_1} + R_{1,2}^3 e^{-2k_2 T_1} - R_{1,2}^5 e^{-4k_2 T_1} + \dots \end{aligned} \quad (9)$$

The  $R_{1,2}$  in Eqs. (8-10) corresponds to the general reflection coefficient of half space (Eq. (6)). Therefore, the general reflection coefficient for a plate of limited thickness can be considered as a modification of that of a half space. The modification is

$$\Delta \tilde{R}_{1,2} = -R_{1,2} e^{-2k_2 T_1} + R_{1,2}^3 e^{-2k_2 T_1} - R_{1,2}^5 e^{-4k_2 T_1} + \dots \quad (10)$$

Eq. (10) shows that the thinner the slab, the larger the modification. The ratio of the change of general reflection coefficient to that of half space is

$$\frac{\Delta \tilde{R}_{1,2}}{R_{1,2}} = -e^{-2k_2 T_1} + R_{1,2}^2 e^{-2k_2 T_1} - R_{1,2}^4 e^{-4k_2 T_1} + \dots \quad (11)$$

If the measurement is conducted at low frequencies,  $R_{1,2}$  is of a small value that  $\frac{\Delta \tilde{R}_{1,2}}{R_{1,2}}$  can be approximated by  $\frac{\Delta \tilde{R}_{1,2}}{R_{1,2}} \approx -e^{-2k_2 T_1}$ . It can be written in the logarithm scale as  $\ln(\Delta \tilde{R}_{1,2}) - \ln(R_{1,2}) = 2k_2 T_1$ .

### ③ Two overlapped slabs

The eddy current measurement of two overlapped slabs can be modelled by 4 regions: regions 1 and 4 are air, regions 2 and 3 are the two overlapped conducting layers. The separation of the signals of the two conductive layers depends significantly on the difference of their electromagnetic properties.

The reflection coefficient showing the reflection between the two conductive regions 2 and 3 is

$$R_{2,3} = \frac{\mu_3 k_2 - \mu_2 k_3}{\mu_3 k_2 + \mu_2 k_3} = \frac{\mu_3^2 k_2^2 - \mu_2^2 k_3^2}{(\mu_3 k_2 + \mu_2 k_3)^2}. \quad (12)$$

Obviously the larger the difference on the electromagnetic properties, the larger the  $R_{2,3}$ , and consequently larger possibility of distinguishing signals from the two slabs.

Because  $\tilde{R}_{3,4} = R_{3,4}$ , the general reflection coefficient  $\tilde{R}_{2,3}$  showing the reflection incorporating the sublayers is written as

$$\tilde{R}_{2,3} = \frac{R_{2,3} + \tilde{R}_{3,4} e^{-2k_3(d_3-d_2)}}{1 + R_{2,3} \tilde{R}_{3,4} e^{-2k_3(d_3-d_2)}} = \frac{R_{2,3} + R_{3,4} e^{-2k_3 T_2}}{1 + R_{2,3} R_{3,4} e^{-2k_3 T_2}} \quad (13)$$

and the general reflection coefficient of the measurement system is

$$\tilde{R}_{1,2} = \frac{R_{1,2} + \tilde{R}_{2,3} e^{-2k_2 T_1}}{1 + R_{1,2} \tilde{R}_{2,3} e^{-2k_2 T_1}}. \quad (14)$$

$\tilde{R}_{2,3}$  indicates the coupling from the lower layer.

### ④ Two air-gap-separated conductive plates

The eddy current measurement of two air-gap-separated plates can be modelled by 5 regions: regions 1, 3 and 5 are air, and region 2 and 4 are conductive slabs, whereas  $R_{2,3} = -R_{1,2}$ ,  $R_{4,5} = -R_{3,4}$ .

$$\tilde{R}_{1,2} = \frac{R_{1,2} + \tilde{R}_{2,3} e^{-2k_2 T_1}}{1 + R_{1,2} \tilde{R}_{2,3} e^{-2k_2 T_1}} \quad (15)$$

The influence of the lower layer shows up in  $\tilde{R}_{2,3}$  that

$$\tilde{R}_{2,3} = \frac{R_{2,3} + \tilde{R}_{3,4} e^{-2k_3 T_2}}{1 + R_{2,3} \tilde{R}_{3,4} e^{-2k_3 T_2}}$$

where  $k_3 = \lambda_0$ ,  $T_2$  is the thickness of the air gap. The influence of the lower layer decreases with the increase of air gap. If the air gap is sufficiently thick,  $e^{-2k_3(d_3-d_2)} \rightarrow 0$ , the influence from the lower layer can be ignored.

### 3. CALCULATION OF REFLECTION COEFFICIENTS AND IMPEDANCES OF LAYERED STRUCTURES

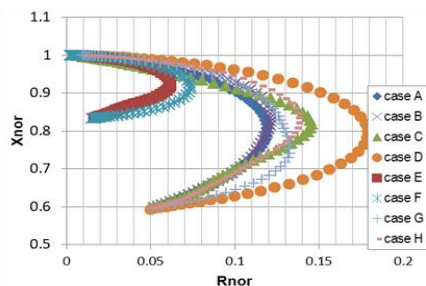
The reflection coefficients and impedance of the cases listed in table 1 are calculated. A coil whose inner, outer and height are respectively 8mm, 10mm and 3mm is assumed for the eddy current measurement. [2,3] showed that the coil (with 0.5mm liftoff) is able to characterize an up to 6mm thick single conductive plate. Therefore, case A is comparable to a half space, case B is to the assessable thickness's upper limit. Aside from case A, the entire thickness (including the liftoff) of each measurement system is less than 6.5mm. The frequency changes from 300Hz to 30kHz, and the assumed test objects' conductivity is 1.4MS/m.

Fig. 3 shows real and imaginary parts of general reflection coefficients of selected cases, where (a) and (b) are for case D, (c) and (d) are for case C, and (e) and (f) are for half space. The differences on the imaginary part of general reflection coefficients are obvious.

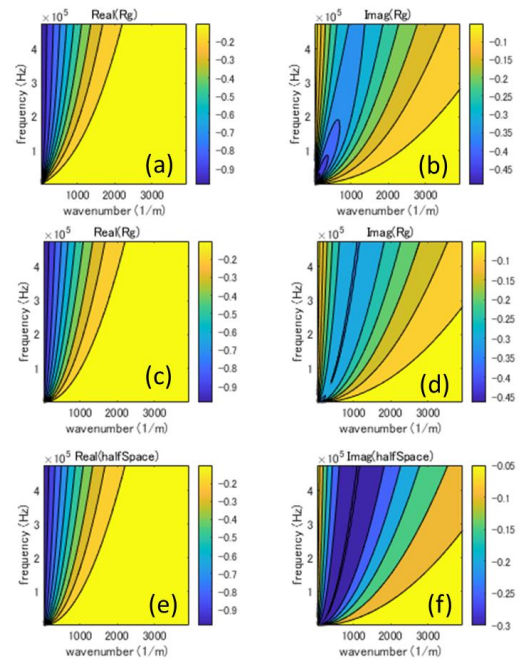
Eq. (3) shows that the impedance of the eddy current measurement is the integrand of the product of shape function and the general reflection coefficient. Fig.4 shows that normalized impedances [2,3]. The difference on liftoff results in the difference on shape function, consequently leads to the impedance difference between case C and E, and difference between case D and F. Nevertheless, reflection coefficient is the essential factor.

**Table 1:** PERCENTAGE OF PAPERS THAT SHOULD BE FORMATTED CORRECTLY

	liftoff (mm)	T1(mm)	T2 (air gap, mm)	T3 (mm)
A	0.5	12	X	X
B	0.5	6	X	X
C	0.5	3	X	X
D	0.5	1	X	X
E	3	3	X	X
F	3	1	X	X
G	0.5	1	1.5	3
H	0.5	3	1.5	1



**FIGURE 4:** NORMALIZED IMPEDANCES.



**FIGURE 3:** GENERAL REFLECTION COEFFICIENTS OF SELECTED CASES. ((a) and (b) for case D, (c) and (d) for case C, and (e) and (f) for half space)

### 4. CONCLUSION

The eddy current measurement of layered structures is analyzed in the view of transmission and reflection of waves between the layers. The contributions of the layers are clarified by the reflection coefficients. The thicknesses are able to be characterized based on the spectrum analysis of reflection coefficients.

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