

## MODELING ULTRASOUND PROPAGATION IN CURVED CFRP COMPOSITES

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### ABSTRACT

Progress is summarized on a computational model for ultrasound transmission in a curved composite laminate. Beam transmission is modeled using a homogenization of the laminate microstructure, by which a quasi-isotropic laminate is represented as a transversely isotropic continuum. The model for the curved laminate thereby has local elastic properties given by the homogenized medium, which rotate with laminate curvature. Beam transmission uses a ray tracing algorithm for curved media. Ongoing work is extracting frequency and propagation direction dependent attenuation coefficients to accurately model scattering losses in the laminate arising from ply interface reflection/refraction that become appreciable at the intermediate wavelengths used for inspection.

Keywords: CFRP composites, Ultrasound Modeling.

### NOMENCLATURE

$\omega$	Radial frequency (rad/sec)
$k$	Spatial frequency (rad/m)
$\theta^{\text{in}}$	Incidence angle
$c_{ij}$	Elastic constants

### 1. INTRODUCTION

Ultrasound propagation in curved CFRP composites presents a modeling challenge due to the elastically heterogeneous anisotropic nature of the curved composite laminate. Propagation in the laminate structure displays a substantial wavelength dependence. In a planar quasi-isotropic laminate, propagation at long wavelength (low frequency) behaves as if in a homogeneous transversely isotropic solid, governed by the static quasi-isotropic laminate elastic properties. As wavelength shortens, the heterogeneous laminate structure begins to influence propagation, resulting in a substantial deviation from the transversely isotropic behavior observed in the long wavelength limit. The challenge to be addressed is modeling propagation in the mid-wavelength regime where NDE

inspections are performed so as to adequately simulate this deviation from the transversely isotropic limit, and to do so with a computationally efficient analysis. A further complication is encountered when considering curved laminates, in that the long wavelength propagation behavior is no longer homogeneous transversely isotropic, but rather displays heterogeneity in the form of a position-dependent orientation of the limiting elastic properties which follows the curvature of the laminate. This curvature in long-wavelength elastic property orientation results in propagation along curved ray paths. Modeling in the curved laminate requires treatment of both moderate wavelength laminate interaction and heterogeneous curved ray path propagation. A summary is presented here of ongoing modeling efforts to address these issues in ultrasound transmission in curved CFRP laminates.

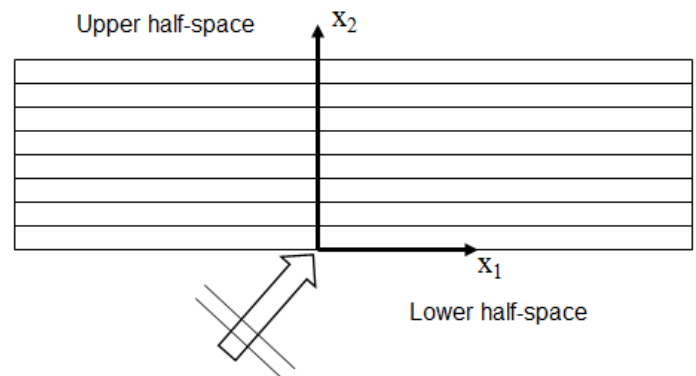


FIGURE 1: PLANE WAVE INCIDENCE ON LAMINATE

### 2. MATERIALS AND METHODS

The approach pursued replaces a planar laminate with an anisotropic homogeneous continuum. Elastic constants for the equivalent continuum are extracted from an exact computational model for plane wave transmission through a planar laminate, as depicted in Figure (1). Each ply of the laminate is assumed to be a transversely isotropic CFRP having elastic properties

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$c_{11}=13.2d9$ ,  $c_{12}=6.42d9$ ,  $c_{33}=103.d9$ ,  $c_{44}=6.16d9$ ,  $c_{13}=5.0d9$ , where  $c_{33}$  corresponds to the fiber direction, and material density of  $1.59\text{kg/m}^3$ . The laminate is laid up in a quasi-isotropic configuration, and is assumed to be sandwiched between two half-spaces comprised of the same CFRP material as the laminate plies. The symmetry direction (fiber direction) of the half-spaces is assumed to be in the  $x_3$  direction (out-of-page). Plane wave incidence is considered with phase propagation direction lying in the  $x_1$ - $x_2$  plane. Due to transverse isotropy, propagation in the half space at this orientation behaves as if in an isotropic half-space. A plane wave of either L, TV, or TH motion is incident on the lower laminate surface, defined to have zero phase at the coordinate origin in Figure (1). Equivalent elastic properties are extracted by observing the phase  $\phi$  of the transmitted L, TV, or TH plane wave on the top surface of the laminate at  $x_1=0$ ,  $x_2=h$ , where  $h$  is the thickness of the laminate

$$\phi = k_2^\alpha(k_1) h \quad (1)$$

$$k_1 = k_\alpha \sin(\theta^{\text{in}}), \alpha = \text{L, TV, TH}$$

where  $\theta^{\text{in}}$  is the plane wave incidence angle in Figure (1). By noting for mode types  $\alpha=\text{L, TV, TH}$  the values of  $\phi$  at  $\theta^{\text{in}}=0$ , denoted  $k_2^{\text{L}0}$ ,  $k_2^{\text{TV}0}$ ,  $k_2^{\text{TH}0}$ , and noting values of  $k_1$  for which  $k_2^\alpha(k_1)=0$ , denoted  $k_1^{\text{L}0}$ ,  $k_1^{\text{TV}0}$ ,  $k_1^{\text{TH}0}$ , it is a straightforward calculation to determine the transversely isotropic elastic constants for the effective medium. Using 1 MHz incidence on a 32 ply 5 mm thick quasi-isotropic layup yields

$$\begin{aligned} c_{11} &= c_{33} = \rho(\omega/k_1^{\text{L}0})^2 = 44.E9 \\ c_{22} &= \rho(\omega/k_2^{\text{L}0})^2 = 13.2E9 \\ c_{44} &= c_{66} = \rho(\omega/k_2^{\text{TV}0})^2 = 4.31E9 \\ c_{55} &= \rho(\omega/k_1^{\text{TH}0})^2 = 12.7E9 \\ c_{13} &= c_{11} - 2c_{55} \end{aligned} \quad (2)$$

The remaining constant  $c_{12}=c_{23}$  cannot be determined in this simple fashion using the configuration of Figure (1). However, it is observed that the propagation scenarios of interest display little dependence on this constant, so a simple volumetric average of this constant over the laminate is employed, yielding  $c_{12}=c_{23}=5.71E9$ . A more rigorous determination can be obtained by fitting to observed values of  $k_2^\alpha$  at other angles of incidence, should this ever become an issue.

## 2.1 Curved Ray Tracing

A beam transmission model is implemented for application to curved media having local elastic properties given by eq.(2). A 2D model of beam transmission is considered here for purpose of demonstration. The beam radiated by a transducer is expressed as a sum of Gaussian beam radiators distributed over the face of the transducer.[1] The beam transmits through the entry surface of a curved CFRP laminate. Starting at the entry surface, the phase front generated by each constituent Gaussian beam radiator is traced through the equivalent curved medium by stepping a small increment in the direction of energy propagation for the mode of interest, noting the perturbation in

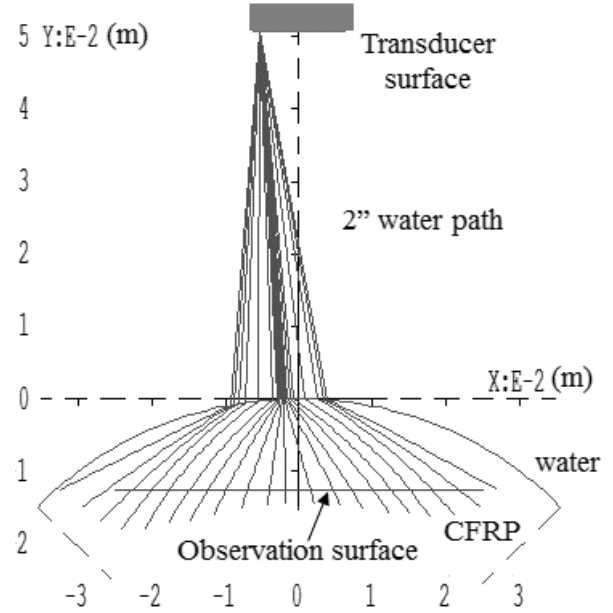
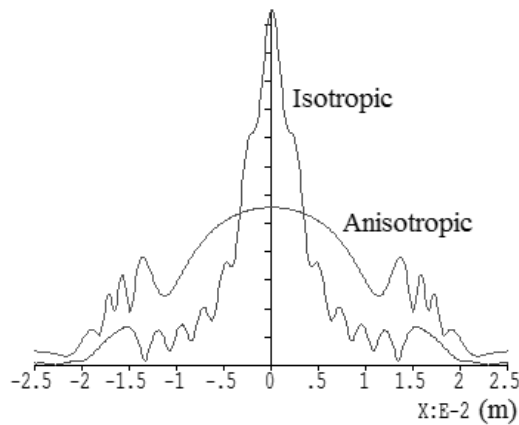


FIGURE 2: RAY TRACING IN CURVED EQUIVALENT CFRP

phase direction arising from the local velocity gradient, computing a corresponding new energy direction, then stepping a subsequent increment in the new energy direction. The field amplitude along the ray is computed from neighboring ray divergence.[2] Results of the ray tracing are shown in Figure (2) for a curved laminate with a 5 cm radius cylindrical entry surface. The local elastic properties are given by Eq.(2), with elastic symmetry locally aligned in the radial direction. Radiation by a 5 MHz 0.5" dia. 5" GFL transducer is represented by a distribution of 7 Gaussian beam radiators over the transducer aperture. The transducer is positioned 2" from the entry surface. Figure (2) plots transmitted QL rays arising from one of the Gaussian beam radiators on the face of the transducer. The field amplitude and phase is evaluated at the intersection of each ray with an internal observation plane. Rays are plotted in Figure (2) for equally spaced rays on the evaluation plane. Note that the ray density at the center of the beam is much greater at the entry surface. This indicates a rapid ray divergence upon transmission, due to a substantially higher wave velocity parallel to the laminate surface. The consequence of this ray divergence is seen in the transmitted beam profiles examined below. The fields evaluated on the observation plane generated by each Gaussian beam radiator are summed to form the transmitted beam generated by the transducer.

Two cases of 5 MHz beam profiles plotted over a 1.25 cm deep internal observation plane are compared in Figure (3). The first case considers a 5 cm radius convex cylindrical interface on an isotropic solid displaying the same longitudinal wave velocity as the perpendicular L-wave velocity of the equivalent CFRP (the  $x_2$  direction if Figure (1)). The second case considers the curved CFRP example depicted in Figure (2). The widening of the beam profile cause by the increased ray divergence upon transmission noted in Figure (2) is clearly evident in Figure (3).



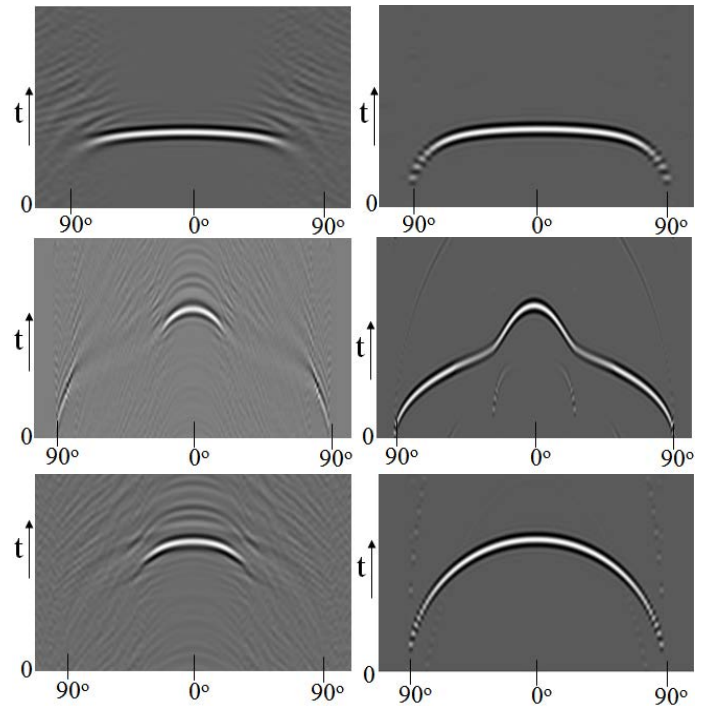
**FIGURE 3: BEAM PROFILE COMPARISON FOR ISOTROPIC AND ANISOTROPIC CURVED ENTRY TRANSMISSION**

## 2.2 Frequency Dependent Properties

Ray transmission in the homogenized continuum of Figure (2) assumes lossless scatter-free transmission along the ray paths shown. In truth, the responses depicted in Figures (2,3) represent the low frequency limit of the laminate transmission properties. As wavelength shortens, reflection/refraction by individual ply interfaces introduces a deviation from the low frequency limit. To understand and incorporate this deviation, the broadband transmission response of the laminate is examined and compared over a range of center frequencies. The study considers the temporal response of a broadband incident plane wave for the configuration of Figure (1).

## 3. RESULTS AND DISCUSSION

Results are shown for plane wave transmission of a 3.5 MHz 100% bandwidth pulse in Figure (4). The pulse arrival is plotted versus incidence angle for L-to-L, TV-to-TV, and TH-to-TH wave transmission, where pulse arrival is observed at the  $x_1=0$ ,  $x_2=h$  position in Figure (1), with increasing time plotted in the upward direction. Critical angles for propagation in the laminate are indicated by '90°'. Transmission through the quasi-isotropic laminate (left) is compared to transmission through the homogenized equivalent medium (right). It is noted in all cases that the homogenized medium transmits at all incidence angles without inter-ply scattering losses. L-to-L transmission is seen in the laminate to resemble the homogenized medium up to a  $\sim 25^\circ$  propagation angle, beyond which a significant amplitude loss occurs, accompanied by an increase in scattering noise. A similar transition occurs in the TV-to-TV and TH-to-TH transmission at  $\sim 15^\circ$  propagation angle. The reduction in transmitted amplitude displayed by the laminate when compared to the homogenized medium can be expressed by an attenuation coefficient that depends on frequency and angle of propagation with respect to the ply boundary. Ongoing work is addressing the extraction of these attenuation coefficients from plane wave transmission results as shown in Figure (4).



**FIGURE 4: 3.5 MHz BROADBAND TRANSMISSION (LEFT) LAMINATE (RIGHT) HOMOGENIZED EQUIVALENT FOR (TOP) L-WAVE (MIDDLE) TV-WAVE (BOTTOM) TH-WAVE**

## 4. CONCLUSION

Progress was summarized on the development of a computational model for ultrasound transmission in a curved composite laminate. Beam transmission is modeled using a homogenization of the laminate structure, by which a quasi-isotropic laminate is represented as a transversely isotropic continuum. The model for the curved laminate thereby has local elastic properties given by the homogenized medium which rotate with laminate curvature. Beam transmission uses a ray tracing algorithm for curved media. Ongoing work is extracting frequency and propagation direction dependent attenuation coefficients to accurately model scattering losses in the laminate arising from ply interface reflection/refraction that become appreciable at the intermediate wavelengths used for inspection.

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