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FOCUSING OF ELASTIC WAVES IN BULK MEDIA USING ADD-ON CONCENTRIC CYLINDRICAL SHELLS

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ABSTRACT

We report the focusing of ultrasonic waves in bulk media by using structures made of concentric cylindrical shells. The methodology of design of proposed structure is based on Huygen's principle and employs guided waves in hollow cylinders. Finite Element (FE) simulations are performed to demonstrate the focusing. The type of waves used and the material selection is discussed along with some of the advantages and limitations. The proposed structures can potentially be an alternate to flat lenses and be used as add-on devices to focus ultrasound waves generated by commercially available unfocused transducers.

Keywords: Focusing, Elastic Waves, Hollow Cylinders.

NOMENCLATURE

v_{cyl}	Velocity of sound in hollow cylinder
v_{Bulk}	Velocity of sound in bulk media
v_{rod}	Velocity of sound in rod
FS	focal spot in bulk media
l	length of hollow cylinder
l_f	distance of focal spot from the top surface of bulk media
r	radius of the hollow cylinder
t_k	constant representing the time of arrival at FS

1. INTRODUCTION

Control of wave propagation in solids has several engineering applications and is of much interest to the Nondestructive Evaluation (NDE) community. In the past two decades, research in the field of artificial structuring of materials for manipulation of wave propagation has gained a lot of momentum. Inspired from the development in the field of electromagnetics, several of these artificial structuring approaches have been demonstrated for acoustic and elastic waves. Of particular interest and relevance to our work here is the use of Gradient Refractive INdex (GRIN) media [1-4].

Typically, a gradient index media is implemented through a base cell repeated in space with variation in one or more of its structural and/or material properties leading to a change in the local velocity of the propagating waves. This has been demonstrated for light as well as sound waves (both bulk as well as surface waves). Researchers, including authors group, have shown the use of graded media in applications such as structural health monitoring (SHM) [5], energy harvesting [6] and altering wavefront curvature [7]. More recently, much thinner structures are being investigated for wave manipulation [8-10]. These are commonly referred to as metasurfaces and are relatively less explored in the case of elastic waves as mode conversion at interfaces poses challenges in the design.

Here, we propose the use of concentric cylindrical shells to cause a phase gradient which, in effect, causes focusing of bulk waves inside a target medium. The outline of the paper is as follows. In section 2, the design of the add-on structure is elucidated with mathematical relations which influence the selection of structural parameters. Results of Finite Element (FE) simulations performed to demonstrate the functioning of proposed design are presented in section 3. Section 4 concludes the paper.

2. DESIGN METHODOLOGY

In order to achieve focusing in the bulk media, we rely on multi-point ultrasonic excitation with a phase difference between various points. The phase difference is achieved by using guided waves in thin cylindrical shells [11,12] which are known to travel at a velocity which depends on the radius and the thickness of the cylindrical shell, apart from the material of the shell.

Figure 1 shows a schematic representing the arrangement of concentric cylindrical shells mounted on a bulk media. Predominantly, the longitudinal wave modes inside the cylinders

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would convert into P-waves in the bulk medium and the flexural wave modes inside the cylinders would generate SV-waves in the bulk medium.

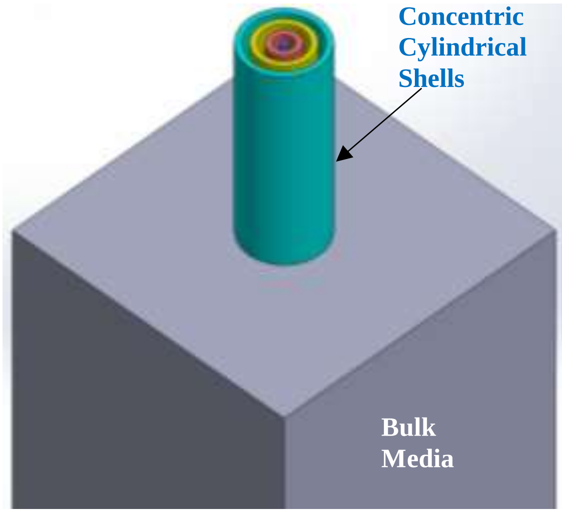

FIGURE 1: Illustrative diagram showing an isometric view of the add-on concentric cylindrical shell structure mounted on the bulk media.

Figure 2 shows the path in which the guided wave, in a cylinder of radius r, travels a path of length l before converting to the corresponding bulk wave. The bulk waves generated at different locations at the interface between the add-on structure and the bulk medium behave as point sources, in accordance with Huygen's principle. These waves constructively interfere at a distance of l_f along the axis of the cylinders.

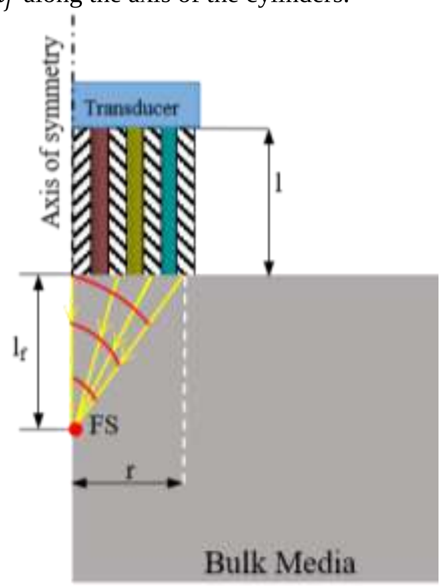

FIGURE 2: Schematic of the cross section of the setup showing geometrical parameters (see details in text).

The phase relationship, between the waves arriving at the interface, that is necessary for the focusing leads to the following relation between various wave velocities and lengths:

$$\frac{l}{v_{rod}} + \frac{l_f}{v_{Bulk}} = \frac{l}{v_{cyl}} + \frac{\sqrt{l_f^2 + r^2}}{v_{Bulk}} = constant(t_k)$$

The constant, t_k, is the time taken for travelling via any of the paths leading to the expected focal spot. In a given target medium, for a fixed focal spot and length of cylinders used to construct the add-on structure, the velocity of wave required within the cylinder is only a function of the radial location.

$$v_{cyl} = \frac{l}{t_k - \frac{\sqrt{l_f^2 + r^2}}{v_{Bulk}}} = f(r)$$

Using the above relation, an estimate of required velocity at different locations can be calculated. The parameters l = 50 mm and l_f = 50 mm were chosen. For focusing, the waves in the peripheral cylinder will need to travel at higher speed and those in the innermost cylinder will be slowest. For the purpose of demonstration, we simulated the propagation of 1 MHz waves in Aluminum (ρ=2700 kg/m^3, v_P=6320 m/s, v_{SV}=3130 m/s). Guided waves with L(0,2) mode in hollow Aluminum cylinders were considered for generation of bulk P-waves. We used Disperse [13] to identify the velocity of these guided waves for cylinders of different radii and thicknesses. Full wave simulations were then carried out in a commercial FE solver [14].

3. RESULTS AND DISCUSSION

Four cylinders were modeled as the constructed add-on structure. The innermost cylinder was taken as a solid rod of 1 mm radius. Figure 3 shows the snapshots of displacement along the wave propagation direction at different time instances. At the expected time of arrival t_k, evaluated to be 18.32 μs in this case study, the high amplitude indicates constructive interference. The wave can be observed to be converging before and diverging after the focal spot.

The design is sensitive to the frequency of operation and the structural parameters through the dispersion characteristics of the guided waves used. A fixed set of radii for positioning the cylinders would decide the thickness of each cylinder. Alternatively, a fixed thickness could, in theory, be used for all cylinders which would then demand the determination of appropriate radii of such cylinders based on the requirement of velocity variation. The study of such varied configurations of the add-on structure and its influence on the focal zone is underway at our research group.

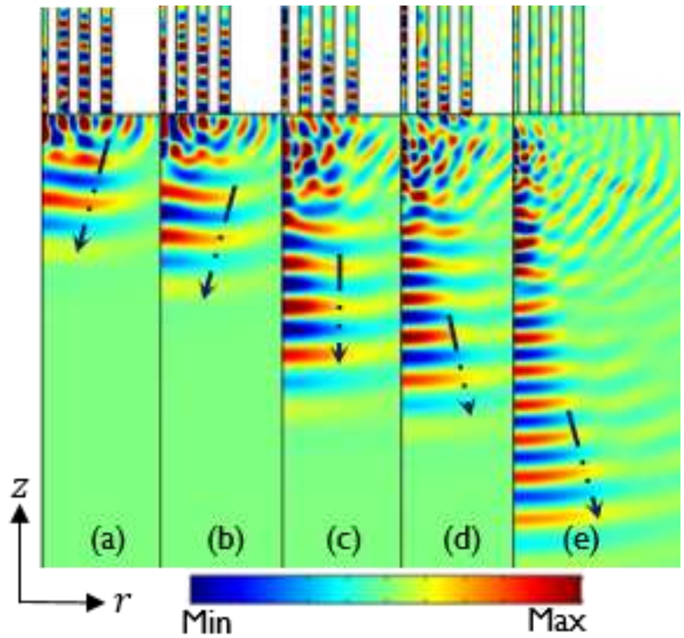

FIGURE 3: Snapshots of displacement in z – direction at different instants of simulation time (in μs) (a) 14.58 (b) 15.62 (c) 18.32 (d) 19.79 (e) 23.9, showing the focusing at 18.32 μs. The arrows indicate the direction normal to the propagating wavefront.

4. CONCLUSION

Add-on structures made of concentric cylindrical shells have been proposed as being capable of focusing ultrasound waves generated by a commercially available unfocused transducer. The methodology of material and structural parameter selection has been discussed. Although the design is frequency dependent and constrained by the volume that the add-on structure can occupy, it is very simple and easy to realize. Numerical simulations indicate the feasibility of the proposed design. In near future, the authors intend to extend the design space and validate the design through experiments.

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