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# METAMODEL-BASED UNCERTAINTY PROPAGATION FOR MODEL-ASSISTED PROBABILITY OF DETECTION: AN OVERVIEW OF RECENT ADVANCES

EXTENDED ABSTRACT

**Xiaosong Du, Leifur Leifsson[1], Jethro Nagawkar**
Department of Aerospace Engineering,
Iowa State University, Ames, IA

**Praveen Gurrala, Jiming Song**
Department of Electrical and Computer Engineering,
Iowa State University, Ames, IA

**William Meeker**
Department of Statistics
Iowa State University, Ames, IA

**Ronald Roberts**
Center for Nondestructive Evaluation
Iowa State University, Ames, IA

## ABSTRACT

*Model-assisted probability of detection (MAPOD) is the key metric for reliability analysis of nondestructive testing (NDT) systems. Fast metamodeling techniques advances the MAPOD process by efficiently capturing the physics and then largely reducing the number of physics-based model evaluations. This work presents the MAPOD analysis through the polynomial chaos-based Kriging (PCKriging) metamodel. In particular, the proposed PCKriging approach is demonstrated on an analytical function and an ultrasonic testing benchmark case and compared against the current state-of-the-art metamodels. Preliminary results in this work show that the PCKriging is capable of reducing the training cost by two to four times fewer than the current state of the art.*

Keywords: nondestructive testing, metamodeling, model-assisted probability of detection, polynomial chaos-based Kriging.

## 1. INTRODUCTION

Model-assisted probability of detection (MAPOD) [1] is the key metric of checking the reliability of the nondestructive testing (NDT) systems. “Model-assisted” refers that the MAPOD analysis relies on physics-based simulation models. On one hand, MAPOD advances the original probability of detection (POD) by reducing experimental budgets. On the other hand, a large number of computationally intensive physics-based model evaluations are typically required for MAPOD analysis.

Metamodel-based MAPOD analysis save the computational cost by using fast and accurate metamodel representing physics information in lieu of computational expensive physics-based models. Kriging interpolation [2] has been commonly used for MAPOD analysis. Least angle regression (LARS)-based polynomial chaos expansion (PCE) [3] introduced by the authors’ prior work show outstanding performance in this themed topic. Moreover, Cokriging multifidelity metamodeling [4], first introduced into MAPOD analysis by the authors, also is promising when multifidelity models are available.

This work assumes that only one level of physics-based model is available. In particular, the polynomial chaos-based Kriging (PCKriging) is introduced and compared against the commonly used Kriging interpolation and the LARS-based PCE.

This paper is organized as follows. Section 2 describes the metamodeling approaches, including Kriging, PCE and the proposed PCKriging. Section 3 demonstrates the PCKriging metamodel on analytical function and MAPOD analysis of benchmark ultrasonic testing case. The paper ends with conclusion and suggestions of future work.

## 2. UNCERTAINTY PROPAGATION VIA META-MODELING

This section provides detailed description of Kriging and PCE metamodeling methods as the benchmarking approaches and the proposed PCKriging method in this work.

### 2.1 Sampling

Sampling is an iterative process of generating parameter values based on corresponding pre-set probabilistic density functions. Monte Carlo sampling (MCS) [5] and Latin Hypercube sampling (LHS) [5] are the sampling tools in this

[1] Contact author: leifur@iastate.edu

work. In particular, MCS is used for generating testing data set while LHS is used for training data set.

### 2.1 Kriging Interpolation

Kriging interpolation [6] has the generalized formula

$$M^{KR}(\mathbf{X}) = \mathbf{f}^T(\mathbf{X})\boldsymbol{\beta} + Z(\mathbf{X}), \tag{1}$$

where $\mathbf{X} \in \mathbb{R}^m$, $\mathbf{f}(\mathbf{X}) = [f_0(\mathbf{X}), \ldots, f_{p-1}(\mathbf{X})]^T \in \mathbb{R}^p$ is defined with a set of the regression basis functions, $\boldsymbol{\beta} = [\beta_0(\mathbf{X}), \ldots, \beta_{p-1}(\mathbf{X})]^T \in \mathbb{R}^p$ denotes the vector of the corresponding coefficients, and $Z(\mathbf{X})$ denotes a stationary random process with zero mean, variance and nonzero covariance. In this work, Gaussian exponential correlation function is adopted, thus the nonzero covariance is of the form

$$Cov[Z(\mathbf{X}), Z(\mathbf{X}')] = \sigma^2 \exp\left[-\sum_{k=1}^{m} \theta_k |X_k - X_k'|^2\right], \tag{2}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_m]^T$, denote the vectors of unknown hyper model parameters to be tuned. Further derivation gives the Kriging predictor $\hat{y}(\mathbf{X})$ for any untried $\mathbf{X}$ as follows

$$\hat{y}(\mathbf{X}) = \mathbf{f}^T\boldsymbol{\beta} + \mathbf{r}^T(\mathbf{X})\mathbf{R}^{-1}(\mathbf{y}_{\mathrm{tr}} - \boldsymbol{\beta}\mathbf{f}), \tag{3}$$

where $\mathbf{y}_{\mathrm{tr}}$ is the observations of training data.

### 2.2 Polynomial Chaos Expansion

PCE [7] metamodel has the generalized formula

$$M^{PC}(\mathbf{X}) \approx \sum_{i=1}^{P} \alpha_i \boldsymbol{\Psi}_i(\mathbf{X}) + \boldsymbol{\varepsilon}, \tag{4}$$

where $\mathbf{X} \in \mathbb{R}^{\mathrm{m}}$ is a vector with random independent components, described by a probability density function $f_{\mathbf{X}}$, $i$ is the index of $i$th polynomial term, $\boldsymbol{\Psi}$ is multivariate polynomial basis, and $\alpha$ is corresponding coefficient of basis function, $\boldsymbol{\varepsilon}$ is the residual between the observations and the PCE predictions, $P$ has the following formula

$$P = \frac{(p+n)!}{p!n!}, \tag{5}$$

where $p$ is the required order of PCE, and $n$ is the total number of random variables.

To solve for the unknown parameters, LARS [7] method is used to minimize the $\boldsymbol{\varepsilon}$ in Eqn. (4)

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} (\mathbf{y}_{\mathrm{tr}} - \sum_{i=1}^{P} \alpha_i \boldsymbol{\Psi}_i(\mathbf{X}) + \lambda \| \boldsymbol{\alpha} \|_1), \tag{6}$$

where $\lambda$ is a scale parameter, $\|*\|_1$ is $L_1$ norm to favor low-rank solutions.

### 2.3 Polynomial Chaos-Based Kriging

The proposed PCKriging [8] metamodeling method is constructed based the afore-mentioned Kriging and PCE metamodels. The general process is summarized as follows

(1) construct a LARS-based PCE model first on training data
(2) use the orthogonal polynomial bases selected by LARS as trend function terms of Kriging interpolation,
(3) construct the Kriging metamodel based on Step 2,
(4) once all unknown parameters are determined, predictor is set up in the same way as conventional Kriging.

## 3. NUMERICAL EXAMPLES

This section demonstrates the proposed PCKriging metamodeling method on an analytical short column case and MAPOD analysis of an ultrasonic benchmark problem.

### 3.1 Analytical Function

The short column function models a structural column as

$$f(\mathbf{x}) = 1 - \frac{4M}{bh^2Y} - \frac{P^2}{b^2h^2Y^2}, \tag{8}$$

where $b$ is the width of the cross section and equals 5 mm, $h$ is the depth of the cross section and equals 15 mm, $Y$, $M$ and $P$ are the uncertain parameters in this case and $Y \sim LogN(5, 0.5)$ MPa is the yield stress, $M \sim N(2{,}000, 400)$ MNm is the bending moment, and $P \sim N(500, 100)$ MPa is the axial force.

Results are given in Fig. 1. The root mean squared error is targeted at 1% of standard deviation of testing points ($1\%\sigma_{\mathrm{testing}}$). PCKriging takes only 70 training points, while PCE both need 120 and Kriging needs 1200.

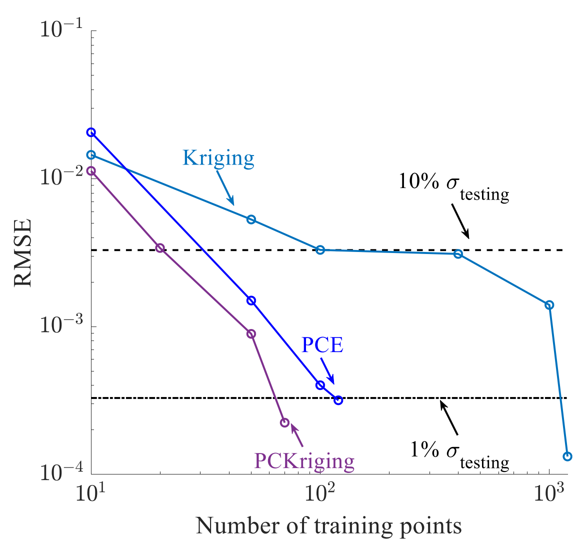


**FIGURE 1:** METAMODELING SETUP.

 

### 3.2 Ultrasonic Benchmark Case

The setup of the ultrasonic benchmark case is given in Fig.2. In this work, the probe angle, $\theta$, and the probe x location, $x$, are considered as uncertain, with normal $N(0, 1)$ deg, and uniform $U(0, 1)$ mm distributions, respectively.

Comparison plots of the root mean squared error (RMSE) is shown in Fig. 3. To reach $1\%\sigma_{\text{testing}}$, PCKriging needs 50 HF training points, while PCE needs around 200, and Kriging needs more than 300. The POD curve is generated and shown in Fig. 4.

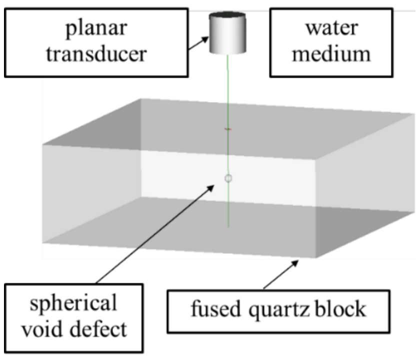


**FIGURE 2:** SETUP OF ULTRASONIC BENCHMARK CASE.

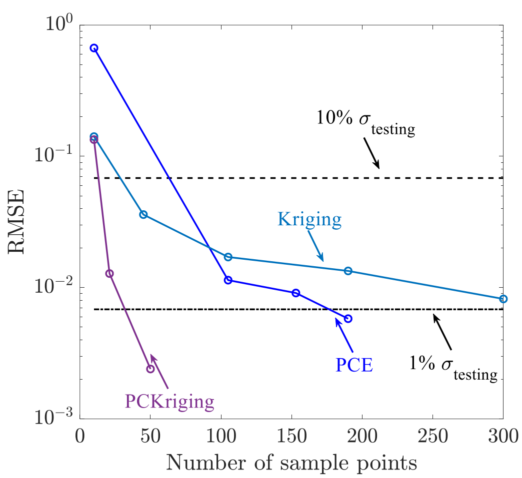


**FIGURE 3:** COMPARISON OF METAMODELING.

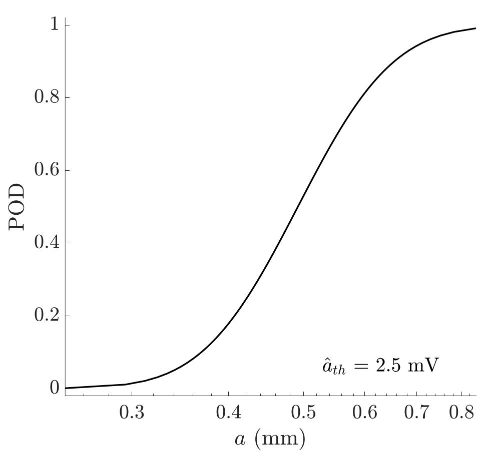


**FIGURE 4:** PCKRIGING-BASED POD CURVE.

## 4. CONCLUSION

This work presents the PCKriging metamodel on analytical function and MAPOD analysis of an ultrasonic benchmark case. The advantage of PCKriging over the current state-of-the-art metamodels in terms of training sample cost is promising. The full paper will show several more UT test cases.


## ACKNOWLEDGEMENTS

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