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MODEL-ASSISTED RELIABILITY ANALYSIS OF NONDESTRUCTIVE TESTING SYSTEMS USING POLYNOMIAL CHAOS-BASED COKRIGING

EXTENDED ABSTRACT

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ABSTRACT

Multidelity modeling representing the high-fidelity physics is a new technique introduced into nondestructive testing (NDT) systems. This work proposes a novel multifidelity modeling method combining polynomial chaos expansion and Cokriging. Specifically, the least angle regression method is used to select the most correlated orthogonal polynomial of the training data and subsequentially used bases as the trend functions of Cokriging model. This proposed polynomial chaos-based Cokriging (PC-Cokriging) is demonstrated for model-assisted probability of detection of NDT systems. The approach is demonstrated on an analytical test case, as well as on an ultrasonic testing case. The results show that PC-Cokriging is capable of reducing the training sample cost to around half of the training cost for the conventional Cokriging method.

Keywords: nondestructive testing, multfidelity modeling, model-assisted probability of detection, polynomial chaos-based Cokriging.

1. INTRODUCTION

Model-assisted probability of detection (MAPOD) [1, 2] is the key metric for assessing the reliability of nondestructive testing (NDT) systems. MAPOD advances the originally proposed probability of detection (POD) by incorporating information from physics-based models and reducing the experimental budget.

The core of MAPOD analysis is the propagation [3] of the variability within random input parameters into the model responses through Monte Carlo (MC)-based computational model evaluations. This MC-based variability propagation process typically requires a large amount of model evaluations, which can limit the application of MAPOD analysis when using accurate physics-based models.

Metamodelin is the process of generating computationally efficient model and capturing the physics information in lieu of time-consuming physics-based simulation models. Data-fit metamodels, including Kriging [4] and polynomial chaos expansions (PCE) [5], have been utilized for fast MAPOD analysis. Multifidelity metamodeling is also introduced for the first time into NDT systems by the author's prior work [6]. The reduction on computational cost of MAPOD using metamodeling and multifidelity methods is promising.

This work proposes a novel multifidelity metamodel, polynomial chaos-based Cokriging (PC-Cokriging), for further saving on training sample cost. The approach is compared against the current state-of-the-art metamodels using benchmark case studies.

This paper is organized as follows. Section 2 describes the key methodologies used in this work. Section 3 demonstrates the proposed PC-Cokriging method on an analytical function and a MAPOD case. The paper ends with conclusion.

2. METAMODELING METHODS

This section provides detailed description of the state-of-the-art metamodels and the proposed PC-Cokriging method.

2.1 Polynomial Chaos Expansion

PCE metamodel has a generalized formula as follows [7]

$$M(\mathbf{X}) \approx \sum_{i=1}^{P} \alpha_i \mathbf{\Psi}_i(\mathbf{X}), \tag{2}$$

where $\mathbf{X} \in \mathbb{R}^m$ is a vector with random independent components, described by a probability density function f_X, i is the index of ith polynomial term, $\mathbf{\Psi}$ is multivariate polynomial basis, and α is corresponding coefficient of basis function, P has the following formula

$$P = \frac{(p+n)!}{p!n!}, \tag{3}$$

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where p is the required order of PCE, and n is the total number of random variables.

2.2 Kriging Interpolation

Kriging interpolation [4] (also known as Gaussian random process) has the generalized formula as follows

$$y(\mathbf{X}) = \mathbf{f}^T(\mathbf{X})\boldsymbol{\beta} + Z(\mathbf{X}), \tag{4}$$

where $\mathbf{X} \in \mathbb{R}^m$, $\mathbf{f}(\mathbf{X}) = [f_0(\mathbf{X}), \ldots, f_{p-1}(\mathbf{X})]^T \in \mathbb{R}^p$ is defined with a set of the regression basis functions, $\boldsymbol{\beta} = [\beta_0(\mathbf{X}), \ldots, \beta_{p-1}(\mathbf{X})]^T \in \mathbb{R}^p$ denotes the vector of the corresponding coefficients, and Z(x) denotes a stationary random process with zero mean, variance and nonzero covariance. In this work, Gaussian exponential correlation function is adopted, thus the nonzero covariance

$$Cov[Z(\mathbf{X}), Z(\mathbf{X}')] = \sigma^2 \exp\left[-\sum_{k=1}^{m} \theta_k |X_k - X_k'|^2\right], \tag{5}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_m]^T$, denote the vectors of unknown hyper model parameters to be tuned. Further derivation gives the Kriging predictor $\hat{y}(\mathbf{X})$ for any untried $\mathbf{X}$ as follows

$$\hat{y}(\mathbf{X}) = \mathbf{f}^T\boldsymbol{\beta} + \mathbf{r}^T(\mathbf{X})\mathbf{R}^{-1}(\mathbf{y}_{\text{tr}} - \boldsymbol{\beta}\mathbf{f}), \tag{6}$$

where $\mathbf{y}_{\text{tr}}$ is the observations on training data.

2.3 Cokriging

Cokriging [4] aims at representing high-fidelity (HF) physics information by fusing HF and low-fidelity (LF) information. This work focuses on two layers of models only. The general process of constructing Cokriging is summarized as two main steps: (1) construct a Kriging metamodel on the LF model as described in Section 2.2, (2) construct another Kriging metamodel on the difference between HF and LF model

$$M_{diff}(\mathbf{X}) = M_{HF}(\mathbf{X}) - \rho \cdot M_{LF}(\mathbf{X}), \tag{7}$$

where ρ is an unknown scale parameter, M_{LF} and M_{HF} are LF and HF models, respectively. Step 2 also follows the same process as described in Section 2.2 except that one more unknown parameter (ρ) needs to be considered.

With the two steps ready, the Cokriging predictor follows the same format with Eqn. (6) but all the matrices and terms are in the complex combination of LF and HF training points. Further detail is provided in Forrester et al. [4].

2.4 Polynomial Chaos-Based Cokriging

The proposed PC-Cokriging multifidelity metamodel is constructed based the PCE and Cokriging. In particular, PC-Cokriging uses PCE as the trend function in each layer of Kriging interpolation metamodel. The general process is summarized as follows:

(1) use the orthogonal polynomial bases terms as trend function terms of Kriging interpolation,
(2) construct the Kriging metamodel based on Step 1,
(3) generate the second Kriging metamodel following the same process as Steps 1 and 2 but on the difference,
(4) once all unknown parameters are determined, predictor is set up in the same way as conventional Cokriging.

3. NUMERICAL EXAMPLES

This section demonstrates the proposed PC-Cokriging multifidelity metamodeling method on Currin function and MAPOD analysis of an ultrasonic benchmark problem.

3.1 Analytical Function

The LF and HF Currin functions considered in this work are

$$f_{HF}(\mathbf{x}) = \left[1 - e^{(-1/2x_2)}\right] \frac{2300x_1^3 + 1900x_1^2 + 2092x_1 + 60}{100x_1^3 + 500x_1^2 + 4x_1 + 20}, \tag{8}$$

$$\begin{aligned} f_{LF}(\mathbf{x}) = & 1/4 f_{HF}(x_1 + 0.05, x_2 + 0.05) \\ & + 1/4 f_{HF}(x_1 + 0.05, \max(0, x_2 - 0.05)) \\ & + 1/4 f_{HF}(x_1 - 0.05, x_2 + 0.05) \\ & + 1/4 f_{HF}(x_1 - 0.05, \max(0, x_2 - 0.05)), \end{aligned} \tag{9}$$

where x_1 and x_2 follow $U(0, 1)$.

Metamodeling results are given in Fig. 1. The root mean squared error (RMSE) is targeted at 1% of standard deviation of testing points (1%σ_{testing}). PC-Cokriging needs only 50 training points, while Cokriging and PCE both need 130 and Kriging needs over 200.

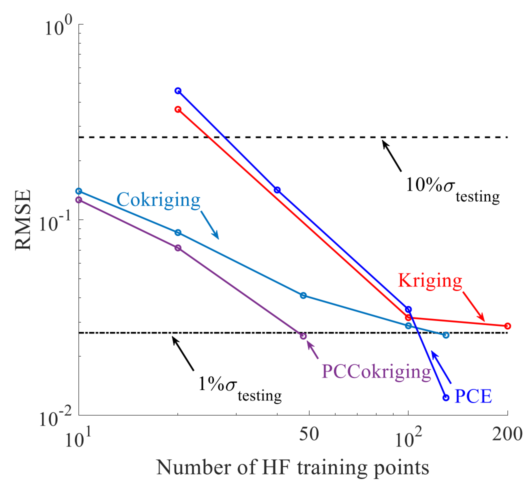

FIGURE 1: METAMODELING SETUP.

3.2 Ultrasonic Benchmark Case

The setup of the ultrasonic benchmark case is given in Fig. 2. In this work, the probe angle, θ, the probe F-number, F, and the probe x location, x, are considered as uncertain, with normal

$N(0,\ 1)$ deg, uniform $U(8,\ 10)$ and uniform $U(0,\ 1)$ mm distributions, respectively.

Comparison plots of root mean squared error is shown in Fig. 3. To reach $1\%\sigma_{testing}$, PC-Cokriging needs 20 HF training points, while Cokriging needs 64, PCE needs 120, and Kriging needs 800. The "ahat vs. a" plots and POD curves are generated and shown in Figs. 4 and 5. The PC-Cokriging results match well with HF plots, while an obvious difference between HF and LF results can be observed.

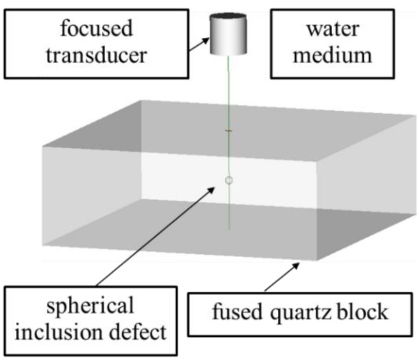

FIGURE 2: SETUP OF ULTRASONIC BENCHMARK CASE.

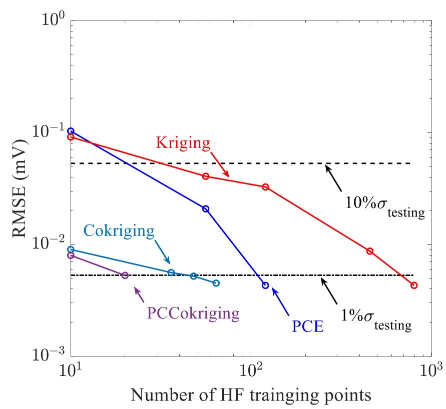

FIGURE 3: METAMODELING SETUP FOR UT CASE.

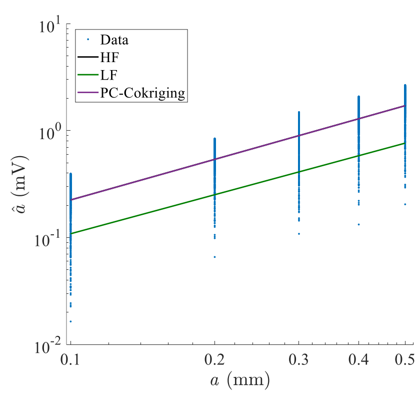

FIGURE 4: "ahat vs. a" LOG-LOG LINEAR REGRESSION.

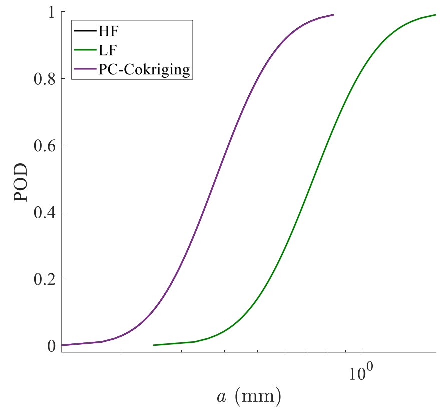

FIGURE 5: POD CURVES FOR THE UT CASE.

4. CONCLUSION

This work introduces the PC-Cokriging multifidelity metamodeling method for MAPOD analysis of NDT systems. In both the analytical function and NDT benchmark cases, the PC-Cokriging outperforms the current state-of-the-art methods. The full paper will present several more NDT cases.

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