

# Restrained Shrinkage in Mildly Reinforced Ultra-High Performance Concrete (UHPC) Structural Members

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## Abstract

Shrinkage is common to all concrete types; however, its effects in Ultra High Performance Concrete (UHPC) can be pronounced. Due to the low water to binder ratio and resulting high paste content, autogenous and drying shrinkage in UHPC can have an important effect on the design of a UHPC member. In prestressed UHPC members, shrinkage results in prestress losses that may be higher than estimated for conventional concrete members. For non-prestressed or mildly reinforced members, shrinkage will result in restraint forces generated in both the reinforcement and the UHPC material. The forces generated in the UHPC can be large which may result in premature cracking and effectively reducing the tensile strain capacity of the UHPC under applied loads if unaccounted for in design. Predicting the restraint forces generated in the mild reinforcement and UHPC require knowledge of early age creep and early age shrinkage of the material, which may be difficult to measure. Therefore, a laboratory test is proposed that can investigate this specific phenomenon and provide this important criterion for design. This paper aims to demonstrate the laboratory test and analysis of the results leading to a proposed design process that a practicing engineer can implement to calculate shrinkage effects in mildly reinforced UHPC structural members.

**Keywords:** UHPC, Restrained shrinkage, Early age creep, Autogenous shrinkage.

## 1 Introduction

Ultra-high-performance concrete (UHPC), like all concrete, is subject to shrinkage during curing and subsequent drying. There are two main types of shrinkage: autogenous and drying shrinkage (De la Varga et al.).

Autogenous shrinkage is a type of shrinkage that occurs due the hydration reactions and will likely have a greater influence UHPC than other concretes due to its low water-to-binder ratio. Autogenous shrinkage is influenced by factors such as the water-to-binder ratio, type and amount of cement used in the mix, as well as the curing conditions (Lura et al.). These influences are more

pronounced in UHPC than conventional concretes due to the constituents and proportions common to UHPC.

Drying shrinkage is common to all concretes and occurs when the water in the concrete evaporates, causing contraction due to reduced pore pressure and dimensional instability. This type of shrinkage is influenced by factors such as the water-to-binder ratio (Huang and Ye), the ambient temperature and humidity during the post curing drying process (Yalçınkaya and Yazıcı), and the size of the concrete structure (Sun et al.).

Both types of shrinkage can lead to cracking and reduced capacity in structures if the structural element cannot contract freely as it shrinks thus generating tensile forces. Two important sources of restrained shrinkage in UHPC members are embedded reinforcement, which will be addressed in this paper, and casting of UHPC against existing conventional concrete members for composite behavior or repair such as in overlays. Both types of restraint may be accounted for in design using the method and testing provided in this paper.

Early age creep and shrinkage will interact in a UHPC member in predictable manner per structural mechanics using an age adjusted effective modulus approach; however, the measurement and separation of these properties can be difficult as they occur simultaneously and at very early age.

Early age shrinkage is dependent upon the datum selected while early age creep requires detailed measurements, setup, and load. Calculating shrinkage stress in UHPC due to embedded reinforcement is an important consideration in the design of structural elements; therefore, the design engineer must be able to accurately assess the impact of this phenomenon in calculating the service and ultimate limit state behavior of a UHPC structural member. This paper will focus on studying the restrained shrinkage in UHPC material and its interaction with early age creep.

## 2 Mechanics

To determine the force in the specimen, the fixed-end shrinkage restraint force should be applied, then the fixed-end shrinkage restraint force should be released onto the composite age-adjusted section. Finally, these two cases should be added. The following is the proposed procedure to determine the forces and strains in the steel and UHPC due to reinforcement restrained shrinkage:

$$\widehat{F}_c = -A_c * \varepsilon_{Free,sh} * \left( \frac{E_c}{1 + \phi_{c,sh}} \right) \quad \text{Equation 1}$$

Where:  $\widehat{F}_c$  = Fixed end shrinkage restraint force,  $A_c$  = Net area of concrete (UHPC),  $\varepsilon_{Free,sh}$  = Free shrinkage strain of UHPC at 28 days,  $E_c$  = 28-day modulus of elasticity of UHPC per manufacturer,  $\phi_{c,sh}$  = UHPC creep coefficient for shrinkage (To be determined). The released fixed end shrinkage restraint force is equal to the fixed end shrinkage restraint force but in the opposite direction ( $-\widehat{F}_c$ ).

$$F_s = -\widehat{F}_c * \frac{A_s * E_s}{A_s * E_s + A_c * \left( \frac{E_c}{1 + \phi_{c,sh}} \right)} = A_c * \varepsilon_{Free,sh} * \left( \frac{E_c}{1 + \phi_{c,sh}} \right) * \frac{A_s * E_s}{A_s * E_s + A_c * \left( \frac{E_c}{1 + \phi_{c,sh}} \right)} \quad \text{Equation 2}$$

Where  $F_s$  = Force in non-prestressed steel;  $A_s$  = Area of steel;  $E_s$  = Modulus of elasticity of steel;  $\varepsilon_s$  = restrained steel strain, test value (below), or the to-be-designed reinforcement strain in steel at 28 days. Tension strain is indicated by positive strains.

$$\varepsilon_s = \frac{F_s}{A_s * E_s} = A_c * \varepsilon_{Free,sh} * \left( \frac{E_c}{1 + \phi_{c,sh}} \right) * \frac{1}{A_s * E_s + A_c * \left( \frac{E_c}{1 + \phi_{c,sh}} \right)} \quad \text{Equation 3}$$

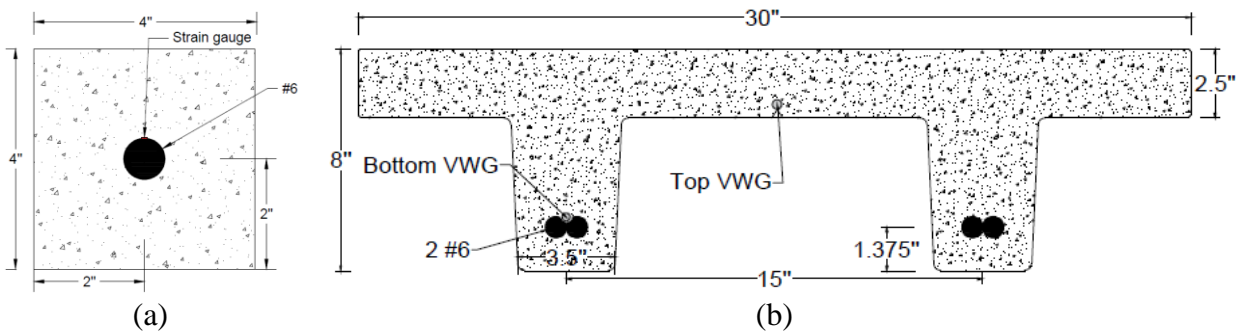
Where  $\varepsilon_s$  = Strain in non-prestressed steel. Solve **Error! Reference source not found.** for  $\phi_{c,sh}$ :

$$\phi_{c,sh} = \frac{-E_c * A_c * \varepsilon_s + E_c * A_c * \varepsilon_{Free,sh} - E_s * A_s * \varepsilon_s}{E_s * A_s * \varepsilon_s} \quad \text{Equation 4}$$

Autogenous shrinkage occurs almost entirely at early age, with drying shrinkage being a much less important effect with UHPC. Thus early restraint will cause immediate creep, which is very difficult to measure. In this approach shrinkage effects are calculated in one step at early age, not applying shrinkage case over the long term like is common do with normal concrete loss or camber estimations. This assumption for UHPC gives validity to our small-scale test method and the design values that it produces. These design values, in particular  $\phi_{c,sh}$ , can be used for both conventionally reinforced and prestressed UHPC members.

### 3 Experimental Program, Results, and Analysis

Two specimen types were part of research sponsored by the USDOT/FHWA SBIR program under contract numbers 6913G620P800090 and 693G621C100007 investigating a new UHPC bridge deck solution which was subjected to numerous fatigue cycle regimes. From this work, the Axially Restrained Shrinkage (ARS) test method to determine early creep and restrained shrinkage was established which was then compared to full scale, corrugated multiple stemmed deck elements (Figure 1). The UHPC material used in the study was supplied by Cor-Tuf, a proprietary UHPC supplier registered with multiple states and federal agencies in the US with 2% steel fiber by volume fraction in accordance with ASTM A820 (0.2mm x 13 mm).



**Figure 1 (a) ARS Test Detail and (b) Corrugated, stemmed specimen dimensions**

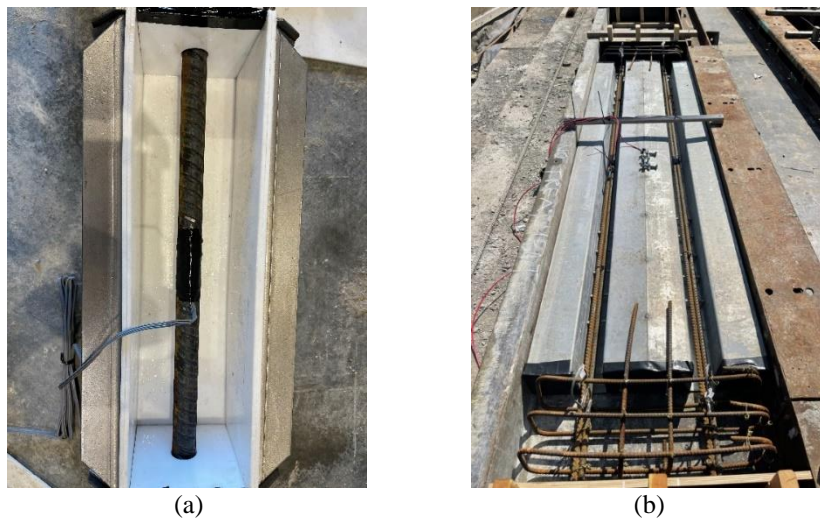
The ARS test method was developed during the period of performance of the sponsored research to assess the combined effects of restrained shrinkage and early-age creep in UHPC. Since completion of the work and prior to writing this paper, the authors did find another who investigated a similar configuration thus validating our approach further, though that work had different goals ((Yoo et al.)). ARS test specimens were 4 in. x 4 in. x 14 in. (10.16 cm x 10.16 cm x 35.56 cm) rectangular prisms. Strain gauges were placed on reinforcing bars along the center of the prism. The preparation of the mold was critical to inhibit bond between the UHPC and the mold and avoid restraint from the form. To eliminate contact between the UHPC mix and the mold, two layers of Polytetrafluoroethylene (PTFE) pads coated with plastic wrap were used on the bottom and each side of the shrinkage mold (as shown in Figure 2a).

The materials and reinforcements used in each shrinkage specimen are presented in Table 1. P in the specimen ID indicates prism specimens, whereas TT indicates double tee specimens. P1 through P3 specimens were cured at 86°F (30°C) to mimic the average steady state temperature of the TT1 through TT8 specimens that were steam cured and stored over a very warm summer month.

**Table 1 Specimens details**

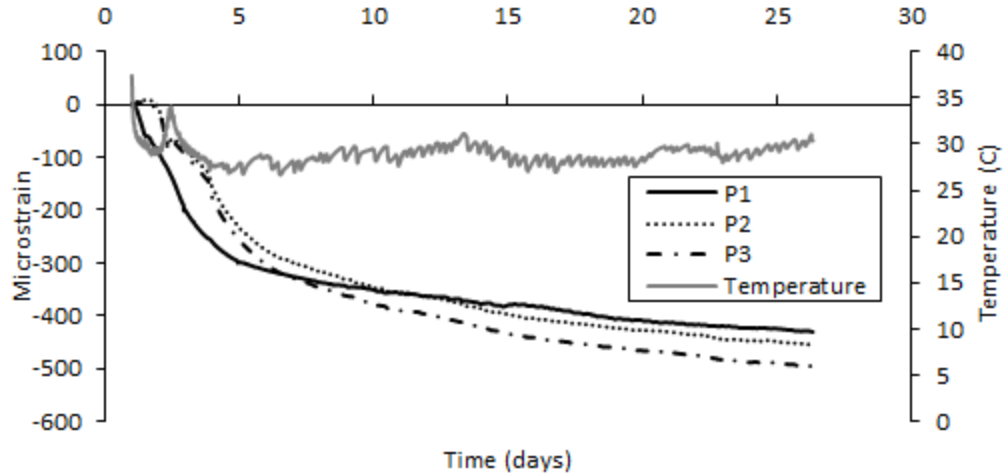
Specimen ID	Configuration	Steel Reinforcement
P1 – P 3	Square prism	1 #6
TT1 – TT3, T5 -TT8	Double Tee	4 #6
TT4	Double Tee	Not Reinforced

Additionally, the shrinkage of eight large-scale UHPC double tee specimens was studied. For each UHPC specimen, two vibrating wire strain gauges (VWSG) were embedded in the panel in a longitudinal direction to measure shrinkage: one VWSG was placed in the top shell (2 inches from the top fiber), and the other was fixed to the reinforcing (centered at 6.625 inches (16.83 cm) from the top fiber) (as shown in Figure 1b).



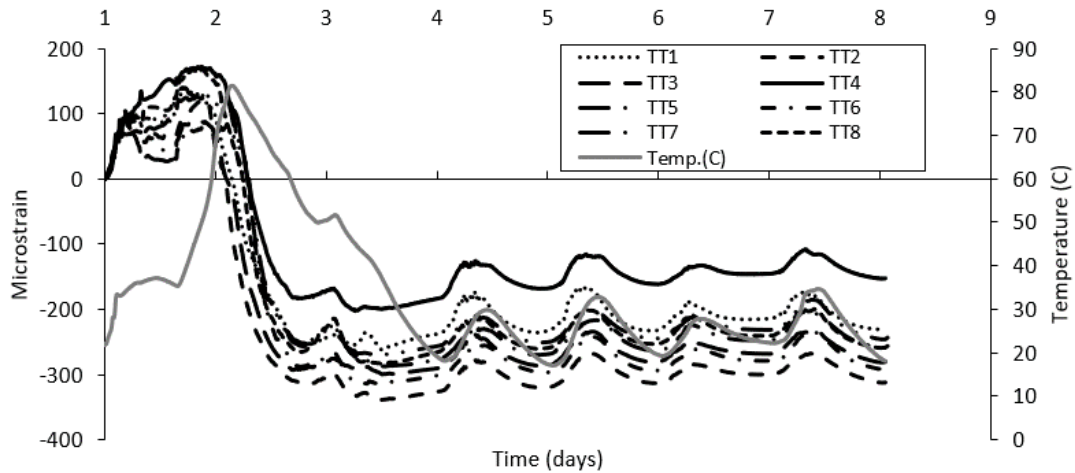
**Figure 2 Shrinkage specimens formwork (a) ARS (b) Double Tee**

The results for ARS specimens P1 through P3 are visualized in Figure 3 and indicate that after curing for 28 days, the average maximum rebar-measured strain caused by the total shrinkage reached  $-450 \mu\epsilon$  after 28 days after which it remained stable. Using Equation 4, with an assumed free shrinkage strain of  $-650 \mu\epsilon$ , modulus of elasticity for Cor-Tuf UHPC of 7100 ksi, (48952.8 MPa) reported by Cor-Tuf, and an average measured strain in the steel of  $-450 \mu\epsilon$  (from strain in Figure 4), revealed an effective UHPC creep coefficient of 2.85, this is considerably higher than then 2.0 normally assumed for long term analyses (e.g. ACI 209).



**Figure 3 Steel microstrain ( $\mu\epsilon$ ) with time at room temperature**

Figure 4 depicts the autogenous shrinkage induced strain readings for large-scale UHPC TT panels, which had approximately  $-200 \mu\epsilon$  after three days following very high temperatures during curing, which may affect a direct comparison. Temperature fluctuates with strain as the concrete temperature changes over time, in particular the diurnal fluctuations after day 4 when the specimen was removed from curing.



**Figure 4 Average VWG readings with time for large-scale specimens**

Table 2 presents the VWG readings for the two (top and bottom) strain gages in the TT deck panels after 100 days, including the difference between after pouring to the time of data stabilization. Note strain gages that had wires inadvertently pulled out during transport are marked with a "-". The average difference in microstrain was  $-426 \mu\epsilon$  and  $-290 \mu\epsilon$  for the top and bottom VWG, respectively for all specimens other than TT4. The average is  $-614 \mu\epsilon$  in the unreinforced TT4 specimen, which when compared to the assumed value from Cor-Tuf of  $-650 \mu\epsilon$  is close. The reinforced specimens exhibited less shrinkage strain than the unreinforced TT4 caused by the restraint of the reinforcement. This finding indicates that the  $-614 \mu\epsilon$  estimate for total (drying plus autogenous) free shrinkage is a reasonable design value as reported by Cor-Tuf.

**Table 2 microstrain readings for mild steel specimens**

Specimen ID	Top VWG ( $\mu\epsilon$ ) Initial Reading	Top VWG ( $\mu\epsilon$ ) Final Reading	Top VWG Total ( $\mu\epsilon$ )	Bottom VWG ( $\mu\epsilon$ ) Initial Reading	Bottom VWG ( $\mu\epsilon$ ) Final Reading	Bottom VWG Total ( $\mu\epsilon$ )
TT1	3205	2683	-521	2621.96	2458	-163
TT2	3233	-	-	3182.86	2948	-234
TT3	3273	-	-	3283.46	3050.	-232
TT4	3223	2588	-635	3183.37	3588	-594
TT5	3179	2733	-445	3219.11	2911	-308
TT6	3216	2762	-453	3233.56	2973	-260
TT7	3141	2764	-377	3270.65	2969	-301
TT8	3138	2797	-341	3254.75	3024	-230

## 4 Examples

### 4.1 Small Scale Test Specimen Example

The small-scale test specimen is a UHPC prism identical to P1, P2, and P3 specimen mentioned in Table 1. Figure 1 depicts the specimen dimensions. The area of steel rebar =  $A_s = 0.44 \text{ in}^2 (2.84 \text{ cm}^2)$ , the net area of concrete =  $A_{c,net} = (4)^2 - 0.44 = 15.56 \text{ in}^2 (100.4 \text{ cm}^2)$ , the modulus of elasticity for steel =  $E_s = 29000 \text{ ksi} (200000 \text{ MPa})$ , and for UHPC =  $E_c = 7100 \text{ ksi} (48952.8 \text{ MPa})$ . The total free shrinkage =  $\epsilon_{Free,sh} = -650 \mu\epsilon$ . The strain in the steel rebar (measured during the experimental tests) =  $\epsilon_s = -450 \mu\epsilon$ . Using **Error! Reference source not found.**, the UHPC creep coefficient for shrinkage =  $\phi_{c,sh} = 2.85$ . Early age creep is a difficult value to obtain through other tests accurately but can be estimated from this type of specimen. Additional experimental investigation is needed to determine if this value is broadly descriptive of the early age or through 28-day effective creep coefficient for a given UHPC. This value is only valid for design when used with assumed  $E_c$  and  $\epsilon_{Free,sh}$  values shown above which would need to be determined through testing for each UHPC. Based on the variability of these properties, we elected to round this value up to 3.0 to overestimate the effects of early age shrinkage in the example that follows.

### 4.2 Large Scale Test Specimen Example

This example explains the procedure proposed to design for autogenous and drying shrinkage in a flexural member using the values obtained for creep coefficient from the ARS, the TT specimens above is a double tee UHPC deck panel simply supported from ends. Figure 1 depicts the specimen dimensions. The UHPC TT panel properties are:  $E_c = 7100 \text{ ksi} (48952.8 \text{ MPa})$ ,  $\epsilon_{Free,sh} = -650 \mu\epsilon$ ,  $\phi_{c,sh} = 3.0$  (rounded up from previous example), Gross area =  $A_{c,gross} = 116.3 \text{ in}^2 (750.3 \text{ cm}^2)$ , Gross moment of inertia =  $I_{c,gross} = 556.8 \text{ in}^4 (23175.8 \text{ cm}^4)$ , Centroid of gross area =  $\bar{y}_{c,g} = 5.352 \text{ in} (13.6 \text{ cm})$ , UHPC net area =  $A_{c,net} = 114.5 \text{ in}^2 (738.7 \text{ cm}^2)$ , UHPC net moment of inertia =  $I_{c,net} = 527.5 \text{ in}^4 (21956.2 \text{ cm}^4)$ , Centroid of net area =  $\bar{y}_{c,net} = 5.413 \text{ in} (13.75 \text{ cm})$ . The steel reinforcement properties are:  $E_s = 29000 \text{ ksi} (200000 \text{ MPa})$ ,  $A_s = 4 * 0.44 \text{ in}^2 = 1.76 \text{ in}^2 (11.35 \text{ cm}^2)$ , steel distance from bottom fiber =  $\bar{y}_s = 1.375 \text{ in} (3.5 \text{ cm})$ ,  $y_{Top} = 8 \text{ in} (20.32 \text{ cm})$ , and  $y_{Bottom} = 0$ .

Age-adjusted transformed section properties:  $E_T = \frac{E_c}{1+\phi_{c,sh}} = 1775 \text{ ksi} (12238.2 \text{ MPa})$ ,  $A_T = 143.2 \text{ in}^2 (923.87 \text{ cm}^2)$ ,  $I_T = 902.3 \text{ in}^4 (37556.6 \text{ cm}^4)$ ,  $\bar{y}_T = 4.603 \text{ in} (11.7 \text{ cm})$ .

Where  $E_T$  = The age adjusted transformed section modulus,  $A_T$  = Area of the transformed section,  $I_T$  = moment of inertia for Transformed section, and  $\bar{y}_T$  = centroid of transformed section.

Actual strains are the strains that could be measured with a strain gauge, and used to compute deflections. The apparent strains are used to calculate stresses directly from the constitutive law plots. This is extremely important when determining the stress in the UHPC since there is plasticity are inelastic strains from shrinkage, creep, and cracking. These components will be delineated in the following example.

**Case 1:** Fixed end shrinkage restraint force:

$$\hat{F}_c = -A_{c,net} * \varepsilon_{Free,sh} * \left( \frac{E_c}{1+\phi_{c,sh}} \right) = 132.1 \text{ kip} (587.6 \text{ kn})$$

$$\varepsilon_{c,actual}^{Top} = \varepsilon_{c,actual}^{Bottom} = 0 \mu\varepsilon$$

Where  $\varepsilon_{c,actual}^{Top}$  and  $\varepsilon_{c,actual}^{Bottom}$  are the actual strain in the UHPC at top and bottom fibers.

$$f_c^{Top} = f_c^{Bottom} = \frac{\hat{F}_c}{A_{c,net}} = 1.154 \text{ ksi} (7.96 \text{ MPa})$$

Where  $f_c^{Top}$  and  $f_c^{Bottom}$  are the stress in UHPC at top and bottom fibers, respectively.

$$\varepsilon_{c,apparent}^{Top} = \varepsilon_{c,apparent}^{Bottom} = \frac{f_c^{Top}}{E_c} = 163 \mu\varepsilon$$

Where  $\varepsilon_{c,apparent}^{Top}$  and  $\varepsilon_{c,apparent}^{Bottom}$  are the apparent strain in the UHPC at top and bottom fibers, respectively.

$$\varepsilon_s = 0 \mu\varepsilon, \quad f_s = 0 \text{ ksi}$$

Where  $\varepsilon_s$  = The strain in the steel rebar and  $f_s$  = The stress in the steel rebar.

**Case 2:** Release fixed end shrinkage restraint force onto transformed section:

$$-\hat{F}_c = -132.1 \text{ kip} (587.6 \text{ kn})$$

$$M = -\hat{F}_c(\bar{y}_T - \bar{y}_{c,net}) = 107.1 \text{ kip.in} (1210 \text{ kn.cm})$$

Where M = transformed section released moment

$$f_c^{Top} = \frac{-\hat{F}_c}{A_T} + \frac{M(\bar{y}_T - \bar{y}_{Top})}{I_T} = -1.326 \text{ ksi} (-9.14 \text{ MPa})$$

$$f_c^{Bottom} = \frac{-\hat{F}_c}{A_T} + \frac{M(\bar{y}_T - \bar{y}_{Bottom})}{I_T} = -0.376 \text{ ksi} (-2.6 \text{ MPa})$$

$$\varepsilon_{c,actual}^{Top} = \frac{f_c^{Top}}{E_T} = -747 \mu\varepsilon$$

$$\varepsilon_{c,actual}^{Bottom} = \frac{f_c^{Bottom}}{E_T} = -212 \mu\varepsilon$$

$$\varepsilon_{c,apparent}^{Top} = \frac{\varepsilon_{c,actual}^{Top}}{1 + \phi_{c,sh}} = -187 \mu\varepsilon$$

$$\varepsilon_{c,apparent}^{Bottom} = \frac{\varepsilon_{c,actual}^{Bottom}}{1 + \phi_{c,sh}} = -53 \mu\varepsilon$$

$$\varepsilon_{s,actual} = \varepsilon_{s,apparent} = \varepsilon_{c,actual}^{Bottom} + \frac{\bar{y}_s}{\bar{y}_{Top}} (\varepsilon_{c,actual}^{Top} - \varepsilon_{c,actual}^{Bottom}) = -304 \mu\varepsilon$$

$$f_s = \varepsilon_{s,actual} E_s = -8.816 \text{ ksi} (-60.78 \text{ MPa})$$

**Sum of the two cases:**

$$f_c^{Top} = 1.154 \text{ ksi} - 1.326 \text{ ksi} = -0.172 \text{ ksi} (-1.19 \text{ MPa})$$

$$f_c^{Bottom} = 1.154 \text{ ksi} - 0.376 \text{ ksi} = 0.778 \text{ ksi} (5.36 \text{ MPa})$$

$$f_s = 0 \text{ ksi} - 8.816 \text{ ksi} = -8.816 \text{ ksi} (-60.78 \text{ MPa})$$

$$\varepsilon_{s,actual} = 0 \mu\varepsilon - 304 \mu\varepsilon = -304 \mu\varepsilon$$

$$\begin{aligned} \varepsilon_{c,actual}^{Top} &= 0 \mu\varepsilon - 747 \mu\varepsilon = -747 \mu\varepsilon & \varepsilon_{c,actual}^{Bottom} &= 0 \mu\varepsilon - 212 \mu\varepsilon = -212 \mu\varepsilon \\ \varepsilon_{c,apparent}^{Top} &= 163 \mu\varepsilon - 187 \mu\varepsilon = -24 \mu\varepsilon & \varepsilon_{c,apparent}^{Bottom} &= 163 \mu\varepsilon - 53 \mu\varepsilon = 110 \mu\varepsilon \end{aligned}$$

$\varepsilon_{c,apparent}^{Top}$  is in linear elastic range,  $\varepsilon_{c,apparent}^{Bottom}$  is in linear elastic range, below 1<sup>st</sup> crack which is 141  $\mu\varepsilon$ . Note that 110  $\mu\varepsilon$  is close to 141  $\mu\varepsilon$ , suggesting that some sections could crack under only the restrained shrinkage effect.

To find the true concrete stress, multiply the apparent concrete strain by  $E_c$  for linear elastic range only as shown.

## 5 Conclusion

This paper has presented a test that can be used, when combined with free shrinkage measurements, to estimate the effective creep induced by restrained shrinkage using a sectional analysis and the age-adjusted effective modulus method. This information and the same mechanics-based process can then be used to estimate strains in the UHPC due to restrained shrinkage. In particular early age creep and autogenous shrinkage are experimentally difficult to determine. The ARS experiment allows an easy and near-direct estimation of these properties, though improvements to this process can be done by coupling these results with autogenous and free shrinkage tests, which may improve accuracy. The mechanics described can then be extended to the design of flexural or axially loaded members to estimate UHPC and steel shrinkage induced restraint forces.

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