Probabilistic Numerical Model of Cracking in Ultra-High Performance Fibre Reinforced Concrete (UHPFRC) Beams Subjected to Shear Loading

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Abstract:

The objective of this paper is to verify the validity of the probabilistic explicit cracking model developed for SFRC to simulate the behaviour of a reinforced UHPFRC beam subjected to a bending load leading to shear failure. The relevancy of the model is evaluated through the simulation of the mechanical behaviour of a reinforced UHPFRC beam submitted to shear loading. The parameters of the modelling approach are determined based on the 3-points bending tests performed on beams sawed from the reinforced UHPFRC beam.

The results suggest that the probabilistic explicit cracking model is relevant to analyze the mechanical behaviour of a reinforced UHPFRC beam and provide precise information about the cracking process of this type of material. The good agreement between the experimental result and the numerical simulations suggests that the studied UHPFRC behaves like usual FRC as the tensile strength and the mechanical effect of the fibres may be modelled by a perfectly brittle behaviour and a nonlinear softening behaviour.

Keywords: Ultra-High Performance Fibre Reinforced Concrete; Numerical Model; Shear, Bending, Cracking

1. Introduction

New construction methods and technologies are in constant evolution in order to expand the lifespan of existing and new structures. As part of the new technologies, new advancements in concrete and cement-based products are gradually changing the design and construction worlds. Materials such as ultra-high performance fibre reinforced concrete (UHPFRC) are increasingly used in structural applications. One of the main reasons of its popularity is related to the existence of national and international recommendations for the design of UHPFRC structures.

These recommendations are efficient and conservative for designing simple statically determinate structures at ultimate limit state. However, they do not possess a sufficient physical base to propose relevant solutions for more complex structures such as statically indeterminate structures or more complex loadings as shear or punching situations. Moreover, the control of cracking in the serviceability limit state is one of the main interests of using UHPFRC. Crack control is very important for durability aspects and is certainly a major advantage of UHPFRC in comparison to structures using traditional reinforcement bars. Nowadays, existing design recommendations do not provide sufficient relevant information regarding cracking at the serviceability limit state (crack opening and spacing). In this way, the best approach for designing structures with respect to both safety and sustainable development is the use of finite element analysis.

The objective of this paper is to verify the validity of the probabilistic explicit cracking model developed for SFRC to simulate the behaviour of a reinforced UHPFRC beam subjected to a bending load leading to shear failure. This objective stems from the desire to validate the following hypothesis: UHPFRC having a compressive strength not exceeding 300 MPa and a percentage of fibres not exceeding 3%-volume have a cracking process and a mechanical behaviour in tension comparable to the ones observed in more classical SFRC (Rossi 2008).

The probabilistic explicit cracking model has been in constant development since the late 1980 (Rossi and Richer 1987, Rossi and Wu 1992, Tailhan, Dal Pont *et al.* 2010, Tailhan, Rossi *et al.* 2013, Rossi, Tailhan *et al.* 2014, Rossi, Daviau-Desnoyers *et al.* 2015, Tailhan, Rossi *et al.* 2015). Originally developed to analyze cracking of structures made of ordinary concrete, the model has been enhanced to simulate cracking in fibre reinforced concrete structures. An improvement of this probabilistic explicit cracking model to permit the analysis of steel fibre reinforced concretes (SFRC) cracking was proposed and validated for different type of SFRC with compressive strengths not exceeding 90 MPa. In the validation tests, beams were subjected to bending loads leading to shear or bending failure mechanisms.

2. Probabilistic Explicit Cracking Model of Steel Fibre Reinforced Concrete (SFRC)

The model is based on three main physical evidences that have been observed experimentally:

- Concrete is a heterogeneous material in which the heterogeneities can be modeled through a random spatial distribution of mechanical properties considered as dominant in the cracking process, namely the Young's modulus and the tensile strength (Rossi and Richer 1987).
- Scale effects of concrete cracking are taken into account by the fact that mechanical properties of the material such as the tensile strength depend on the size of the mesh elements chosen for the finite element analysis (Rossi and Richer 1987, Rossi and Wu. 1992). In contrast, the average post-cracking energy is considered independent of scale effects (Rossi 2012).
- Cracking is explicitly treated through the creation of random kinematic discontinuities, which provides access to quantitative information on the cracking state (number of cracks, opening and spacing). Numerically speaking, these cracks are represented by interface elements.

The crack initiation criterion may be summarized as following: the interface element opens when the normal tensile stress at the centre of the interface element reaches a critical value (R_T). The critical value corresponds to the tensile strength of the material which is randomly distributed through a Weibull distribution function (Tailhan, Dal Pont *et al.* 2010). This critical value depends on the total volume of the two volumetric elements interfaced by the considered interface element. This means that the stress and the rigidity of the interface element briefly become equal to zero until the bridging effect of the fibres takes over. The criterion of crack creation is schematically presented in Figure 1.

Therefore, the creation of cracks in the cement matrix is represented by an elastic perfectly brittle behaviour, whereas the bridging effect of the fibres is described by the following modelling approach.

Normal and tangential stresses in the interface element linearly increase with normal and tangential displacements when a “broken” interface element re-opens to take into account the elastic effect of the fibres inside the crack. Physically speaking, the rigidity of the fibres (inside the cracks) is more important in tension than in shear. Thus, the interface element rigidity is considered different for normal and tangential displacements. In 2D, normal and tangential rigidities of the interface element are K_n’ and K_t’ respectively. The post-cracking elastic behaviour exists until it reaches a threshold value, ζ_0, related to the normal displacement (Figure 1). Once this limit value is reached, the mechanical behaviour of the interface element changes. The normal stress is considered as linearly decreasing with the normal displacement in order to take into account the damage of the bond between the concrete and the fibre, and fibre pullout. The decreasing evolution is obtained by a damage model (D is the parameter which characterizes the damage).

Finally, the interface element is considered definitively broken when the normal displacement reaches a threshold value, ζ_c (Figure 1). This value corresponds to the state where the effect of fibres is considered negligible. It is determined from a uniaxial tensile test. At this point, its normal and tangential rigidities are set to zero.

The post-cracking energy dissipated by the bridging effect of the fibres (W) is considered randomly distributed on the mesh elements. The random distribution is a log-normal distribution function with a mean value independent of the mesh elements size (Rossi 2012) and a standard deviation that is due to the heterogeneity of the material increasing as the mesh elements size decreases. To model a given structural element, the distribution function is determined in the following manner:

- The mean value is directly obtained experimentally from a certain number of uniaxial tensile tests on notched specimens, more specifically from the load-crack opening experimental curves.
- The standard deviation, which depends on the mesh elements size, is determined by an inverse analysis approach that consists of simulating the uniaxial tests with different element mesh sizes. As the mean value of the post-cracking energy is known from the experimental results, several numerical simulations are realized for each mesh size to determine the standard deviation that best fits the experimental results. The inverse analysis approach thus allows finding a relation between the standard deviation and the finite element mesh size.

The threshold parameters ζ_0 and ζ_c are determined by an inverse analysis approach to best fit the simplified triangular stress-displacement curve representing the post-cracking energy (Figure 1) to the experimental tensile softening curve.

To conclude this review of the adopted numerical modelling approach, the probabilistic explicit cracking model is considered a deterministic approach with randomly distributed parameters. Following this approach, a simulation presents a single independent response within the possible envelope of responses. Hence, it is necessary to perform a large number of computations to statistically validate the results following a Monte Carlo method. The number of numerical simulations needed is related both to the structural problem and the structural response dispersion.

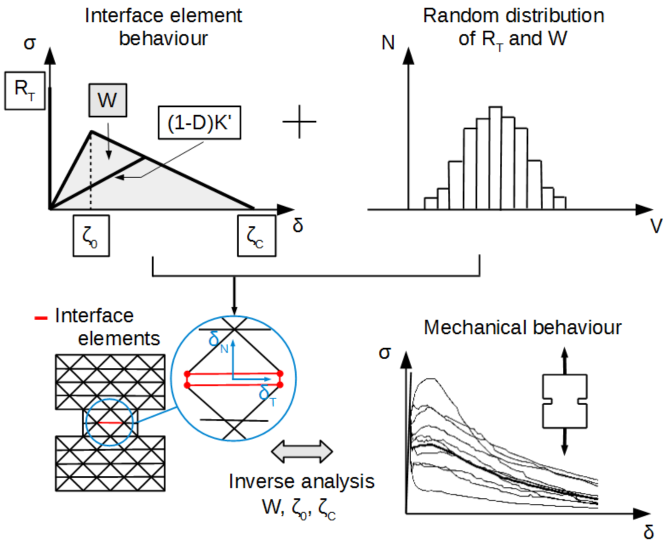

Figure 1. Principles of the explicit probabilistic cracking model of SFRC

3. Numerical Modelling of a reinforced UHPFRC beam

The probabilistic explicit cracking model developed for SFRC has been used to simulate the mechanical behaviour and cracking process of a longitudinally reinforced UHPFRC beam without transverse reinforcement bars. The beam was tested in an experimental campaign performed by Baby (2012) on the tensile behaviour of UHPFRC at the structural

3.1. Experimental Details

The beam, identified as BFUP-F-RC-NS in the experimental campaign, was designed to develop a shear failure mechanism. Therefore, its longitudinal reinforcement was designed to avoid bending failure. The beam's cross section and its reinforcement and the test set-up are respectively presented in Figures 2 and 3. The reinforcement consisted of 5-HA20 and 1-HA25, the latter being placed in the middle of the bottom layer of rebars. Figure 3 shows the boundary conditions of the 4-point bending test, consisting of three roller supports and one pinned. The spans between the loading points and the supports were respectively 480 and 2000 mm.

The UHPFRC mix design used was a self-compacting concrete made of Portland cement, fine sand with a maximum aggregate size of 0.8mm, silica fume, high-range water-reducing admixture, and straight steel fibres with a length of 13 mm and a diameter of 0.2 mm at 2.5 % by volume. The UHPFRC had a 28-day mean compressive strength of 212 MPa, a Young's modulus of 56 GPa and a Poisson's ratio of 0.185.

To evaluate the influence of fibre orientation on the tensile behaviour of the studied UHPFRC, two series of small beams were tested under 3-point bending. The first and second series were made respectively of small beams sawn at a 45° angle and parallel to the reinforced UHPFRC beam length after the test (in parts of the beams not affected by cracking). The beams were 280 mm long, 70 mm high and 60 mm wide, with a 2 mm wide and 8 mm deep notch sawn at mid-span from a single blade stroke. The support span was set to 210 mm. It should be noted that the small beams dimensions were not exactly identical as the beams were sawn from a larger beam. Therefore, the aforementioned dimensions represent the average dimensions of the small beams.

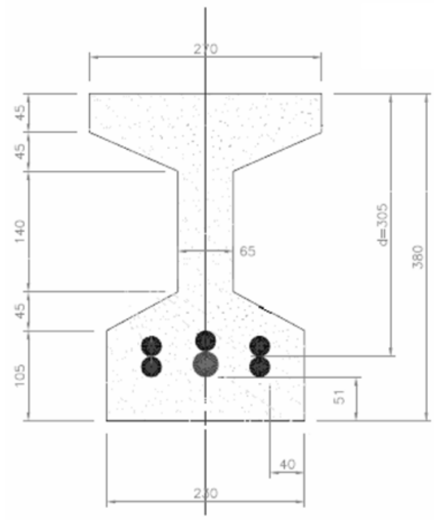

Figure 2. Cross section of the reinforced UHPFRC beam

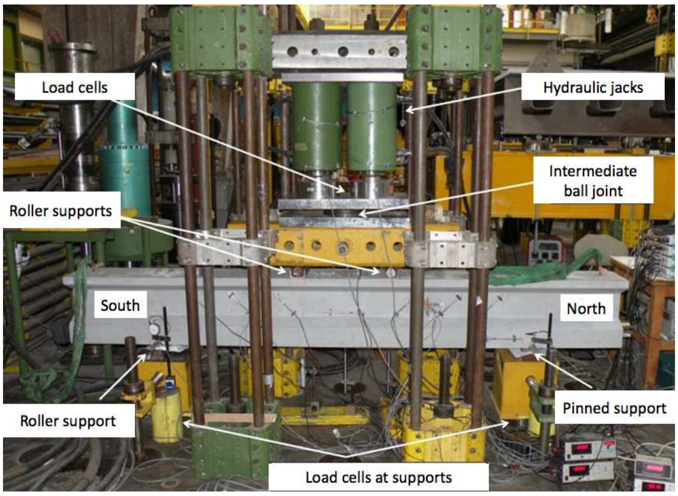

Figure 3. Experimental testing device of the reinforced UHPFRC beam

3.2. Characterization of the tensile behaviour of UHPFRC

Although some general parameters such as the 28-day compressive strength, Young's modulus and Poisson's ratio have been directly determined experimentally, the mechanical characteristics of the tensile behaviour of the studied UHPFRC must be determined by inverse analysis. The inverse analysis approach consists of simulating the 3-point bending tests performed on the small notched beams sawn from the reinforced UHPFRC beam to identify the average tensile strength (R_T) and its standard deviation ($\sigma(R_T)$), the average post-cracking energy (W) and its standard deviation ($\sigma(W)$), and the threshold parameters ζ_0 and ζ_c defining the simplified triangular representing the post-cracking energy.

Generally, the tensile strength and its scattering are determined using the analytical relations presented in Rossi, Wu *et al.* (1994). These analytical functions correspond to the distribution function of the tensile strength with regards to scale effects. Therefore, they mainly depend on the volume of the mesh elements and the maximum aggregate size. In fact, the maximum aggregate size characterizes the heterogeneity of the material, which is in turn at the origin of the scaling effect of the tensile strength of the rupture in tension. However, these analytical equations were obtained for concretes having compressive strengths between 30 and 130 MPa, in which the maximum aggregate size was much larger than the fibres. In contrast, the high volume of fibres combined with their dimensions (compared with the very small diameter of the aggregates used) make the fibres the main source of heterogeneity in the UHPFRCC studied. They induce local stresses concentrations around them and facilitate the initiation of microcracks in the matrix and result in a reduction of the tensile strength. Therefore, the mean tensile strength and its scattering are determined by best fitting the linear elastic segment of the load-deflection curves obtained from the 3-point bending tests.

Figure 4 presents the finite element mesh used to model the 3-point bending tests to identify the mechanical characteristics of the tensile behaviour by inverse analysis. Ten two-dimensional simulations have been performed in plane stresses conditions.

Figure 5 presents the numerical load-crack opening curves obtained from the 3-point bending tests on the notched beams sawed at a 45° angle to the UHPFRC beam length in comparison to the experimental results (as a matter of fact, the structural study being related to the shear beahaviour, only inclined cracks are concerned). The results show a good agreement between the experimental results and the numerical simulations even the numerical simulations underestimate the experimental scattering. This difference may be explained by the fact that the dimensions of the numerical beam were identical for all simulations whereas the dimensions of the experimental beams varied of a few millimeters due to the complexity sawing diagonally the specimens from the reinforced UHPFRC beam.

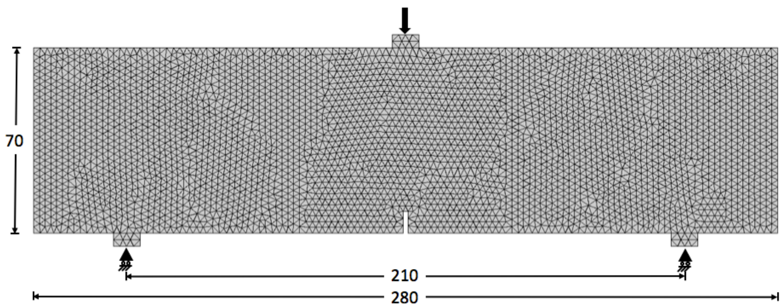

Figure 4. Finite element mesh of the of notched beams tested in 3-point bending

The model parameters obtained through the inverse analysis approach are summarized in Table 1. In addition, the parameters ζ_0 and ζ_c that best fitted the experimental results obtained from the 3-point bending tests are 0.035 mm and 5 mm.

The good agreement between the experimental and numerical load-crack opening responses leads lead suggest that the numerical model originally developed for fibre reinforced concrete seems relevant to model the bending behaviour of UHPFRC.

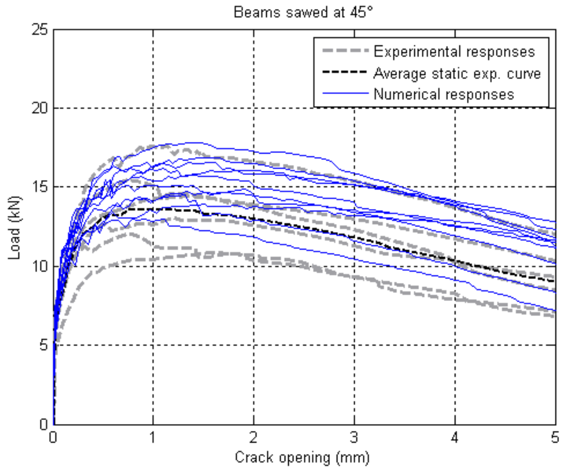

Figure 5. Experimental and numerical load-crack opening curves of the beams sawed at 45°

Table 1. Values of the material parameters used in the numerical model

	R_T [MPa]	$\sigma(R_T)$ [MPa]	W [MPa*mm]	σ(W) [MPa*mm]
Beams sawed at 45	8.43	1.31	60	72

3.3. Numerical Simulation of the Shear Behaviour of a Reinforced UHPFRC Beam

In reinforced concrete beams subjected to shear forces, shear cracks form diagonally, hence at 45° from the length of the beam. The shear failure mechanism may is characterized by the presence of diagonal shear cracks either before or after the formation of flexural cracks. In that respect, the set of parameters that best represent the orientation of the fibres with regards to diagonal cracks corresponds to the parameters identified on the characterization beams sawed at 45°.

Figure 6 presents the finite element mesh used for the reinforced UHPFRC beam simulations. Three two-dimensional simulations have been performed in plane stresses conditions. The reinforcement bars were modelled with linear continuum elements having a height corresponding to the rebar's diameter. Before cracking, the special linear interface elements ensure the continuity of stresses and displacements by using a very high matrix of rigidity. In fact, they are similar to those used for simulating cracking of SFRC without rebars.

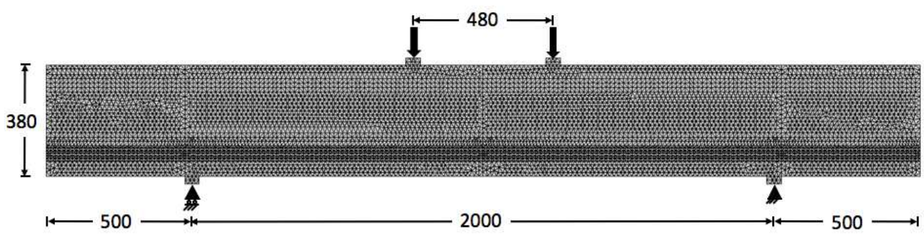

Figure 6. Finite element mesh of the reinforced UHPFRC beam

After cracking, the rebar's effect on the cracks, thus on the interface elements, is considered as a linear elastic behaviour. This behaviour is modelled by setting the normal and tangential components of an interface element's stiffness to be much smaller than before cracking (Tailhan, Rossi *et al.* 2015). The values of these stiffness components are determined through inverse analysis by best fitting the elastic rigidity of the beam.

Figure 7 presents a comparison between the experimental load-deflection curve and the three numerical simulations. The results show a good coherence between the experimental result and the numerical simulations until a deflection of 5 mm, after which the numerical simulations underestimate the experimental behaviour. This may be explained by the numerical identification method for the tensile behaviour of the studied UHPFRC. As mentioned previously, the tensile behaviour of the studied UHPFRC was identified by an inverse analysis approach that consisted on simulating the 3-point bending tests performed on the notched beams sawed from the reinforced UHPFRC beam. Experimentally, it has been observed that multiple cracks initiate at the tip of the notch, therefore not representing the tensile behaviour of the material but rather a structural behaviour in which multiple cracks are sawed by a group of fibres. Therefore, this indirect approach generally underestimates the tensile behaviour of the material due to the stress concentrations occurring at the tip of the notch.

Figure 8 presents an example of a cracking pattern obtained from the numerical simulations at a deflection of 5 mm. Only cracks larger than 100 μm are shown on the figure. The crack pattern shows that several bending cracks were created at the bottom fibre of the beam. Although bending cracks are first created, their propagation is rapidly blocked by the bending reinforcement. These cracks are mainly responsible for the initial loss of flexural rigidity observed at approximately 180 kN, after which the behaviour of the beam is controlled by its shear behaviour. The crack pattern shows that three diagonal cracks with a maximum crack opening of 0.25 mm have formed. Their propagation is controlled by the bridging effect of the fibres as the beam has no stirrups. The cracks opening can be deterrmined from the numerical simulation, but no precise experimental information is accessible to evaluate the accuracy of this prediction.

Figure 9 presents a comparison between the experimental load-deflection curve and the three numerical simulations with the same set of parameters but a 33% increase of the post-cracking energy distribution. The mean post-cracking energy and its standard distribution were respectively increase from 60 to 80 MPa.mm and from 72 to 96 MPa.mm. The results show that the shear behaviour is comparable to the experimental behaviour up to a deflection of 6 mm, after which the model underestimates the shear behaviour of the beam by less than 10%. This confirms that the 3-point bending tests underestimate the post-cracking energy of the tensile behaviour.

To summarize, the results show a good agreement between the experimental result and the numerical simulations both at the global and local scales. Therefore, the results suggest that the studied UHPFRC behaves like usual FRC as the tensile strength and the mechanical effect of the fibres may be modelled by a perfectly brittle behaviour and a nonlinear softening behaviour.

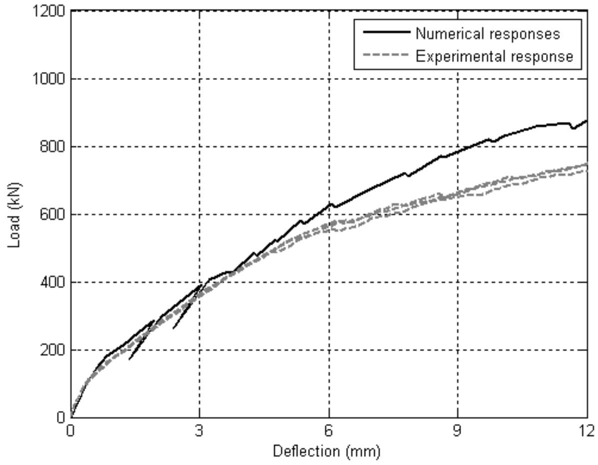

Figure 7. Experimental and numerical load-deflection curves of the reinforced UHPFRC beam

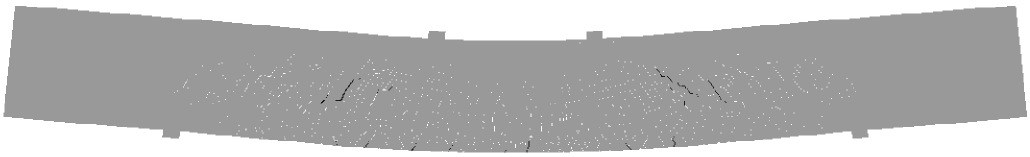

Figure 8. Example of cracking process of the reinforced UHPFRC beam

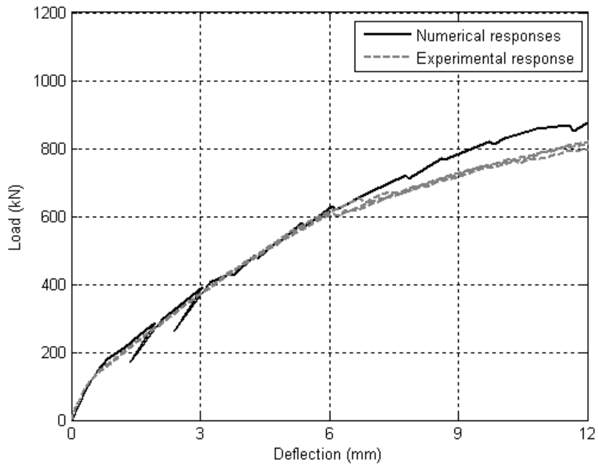

Figure 9. Experimental and numerical load-deflection curves of the reinforced UHPFRC beam with increased post-cracking energy

4. Conclusions

The objective of this paper is to verify the validity of the probabilistic explicit cracking model developed for SFRC to simulate the behaviour of a reinforced UHPFRC beam subjected to a bending load leading to shear failure. The relevancy of the model is evaluated through the simulation of the mechanical behaviour of a reinforced UHPFRC beam submitted to shear loading. The parameters of the modelling approach are determined based on the 3-points bending tests performed on beams sawed from the reinforced UHPFRC beam.

The main conclusion of this work can be summarized as follows.

- The probabilistic explicit cracking model is relevant to analyze the mechanical behaviour of a reinforced UHPFRC beam and provide precise information about the cracking process of this type of material.
- The good agreement between the experimental result and the numerical simulations suggests that the studied UHPFRC behaves like usual FRC as the tensile strength and the mechanical effect of the fibres may be modelled by a perfectly brittle behaviour and a nonlinear softening behaviour.

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